Sections 5.1 and 5.2
Review and Preview
and
Probability Distributions

PROBABILITY DISTRIBUTIONS
This chapter will deal with the construction of probability distributions by combining the methods of Chapters 2 and 3 with the those of Chapter 4.

Probability Distributions will describe what will probably happen instead of what actually did happen.

COMBINING DESCRIPTIVE METHODS AND PROBABILITIES
In this chapter we will construct probability distributions by presenting possible outcomes along with the relative frequencies we expect.
RANDOM VARIABLES AND PROBABILITY DISTRIBUTIONS

- A random variable is a variable (typically represented by $x$) that has a single numerical value, determined by chance, for each outcome of a procedure.
- A probability distribution is a description that gives the probability for each value of a random variable. It is often expressed in the format of a table, formula, or graph.

EXAMPLES

1. Suppose you toss a coin three times. Let $x$ be the total number of heads. Make a table for the probability distribution of $x$.

2. Suppose you throw a pair of dice. Let $x$ be the sum of the numbers on the dice. Make a table for the probability distribution of $x$.

SAMPLE SPACE FOR ROLLING A PAIR OF DICE
DISCRETE AND CONTINUOUS RANDOM VARIABLES

• A **discrete random variable** has a collection of values that is finite or countable. (If there are infinitely many values, the number of values is countable if it is possible to count them individually, such as the number of tosses of a coin before getting tails.)

• A **continuous random variable** has infinitely many values, and the collection of values is not countable. (That is, it is impossible to count the individual items because at least some of them are on a continuous scale.

EXAMPLES

Determine whether the following are discrete or continuous random variables.

1. Let $x$ be the number of cars that travel through McDonald’s drive-through in the next hour.

2. Let $x$ be the speed of the next car that passes a state trooper.

3. Let $x$ be the number of $A$s earned in a section of statistics with 15 students enrolled.

PROBABILITY HISTORGRAM

A **probability histogram** is like a relative frequency histogram with **probabilities** instead of relative frequencies.
EXAMPLES

1. Suppose you toss a coin three times. Let $x$ be the total number of heads. Draw a probability histogram for $x$.

2. Suppose you throw a pair of dice. Let $x$ be the sum of the numbers on the dice. Draw a probability histogram for $x$.

REQUIREMENTS FOR A PROBABILITY DISTRIBUTION

1. There is a numerical random variable $x$ and its values are associated with corresponding probabilities.

2. $\sum P(x) = 1$ where $x$ assumes all possible values.

3. $0 \leq P(x) \leq 1$ for every individual value of $x$.

EXAMPLES

Determine if the following are probability distributions

(a) (b) (c)

<table>
<thead>
<tr>
<th>$x$</th>
<th>$P(x)$</th>
<th>$x$</th>
<th>$P(x)$</th>
<th>$x$</th>
<th>$P(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.20</td>
<td>1</td>
<td>0.20</td>
<td>1</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>0.35</td>
<td>2</td>
<td>0.25</td>
<td>2</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>0.12</td>
<td>3</td>
<td>0.10</td>
<td>3</td>
<td>0.10</td>
</tr>
<tr>
<td>4</td>
<td>0.40</td>
<td>4</td>
<td>0.14</td>
<td>4</td>
<td>0.14</td>
</tr>
<tr>
<td>5</td>
<td>-0.07</td>
<td>5</td>
<td>0.49</td>
<td>5</td>
<td>0.31</td>
</tr>
</tbody>
</table>
MEAN, VARIANCE, AND STANDARD DEVIATION

Mean of a Prob. Dist. \( \mu = \sum [x \cdot P(x)] \)

Variance of a Prob. Dist. \( \sigma^2 = \sum [(x - \mu)^2 \cdot P(x)] \)

\( \sigma^2 = \sum [x^2 \cdot P(x)] - \mu^2 \)

Standard Deviation of a Prob. Dist. \( \sigma = \sqrt{\sum [x^2 \cdot P(x)] - \mu^2} \)

FINDING MEAN, VARIANCE, AND STANDARD DEVIATION WITH TI-83/84 CALCULATOR

1. Enter values for random variable in L1.
2. Enter the probabilities for the random variables in L2.
3. Run “1-VarStat L1, L2”
4. The mean will be \( \bar{x} \). The standard deviation will be \( \sigma \). To get the variance, square \( \sigma \).

TI-84 WITH NEW OPERATING SYSTEM

If you have the TI-84 with the newest operating system, make sure your screen looks like this:

```
1-Var Stats
List:L1
FreqList:L2
Calculate
```
ROUND-OFF RULE FOR $\mu$, $\sigma$, AND $\sigma^2$
Round results by carrying one more decimal place than the number of decimal places used for the random variable $x$.

MINIMUM AND MAXIMUM USUAL VALUES
Recall the Range Rule of Thumb:
minimum usual value = $\mu - 2\sigma$
maximum usual value = $\mu + 2\sigma$

EXAMPLE
Use the range rule of thumb to determine the unusual values for rolling a pair of dice.
IDENTIFYING UNUSUAL RESULTS USING PROBABILITIES

- **Rare Event Rule**: If, under a given assumption the probability of a particular observed event is extremely small, we conclude that the assumption is probably not correct.
- **Unusually High**: \( x \) successes among \( n \) trials is an unusually high number of successes if \( P(\text{at least } x) \) is very small (such as 0.05 or less).
- **Unusually Low**: \( x \) successes among \( n \) trials is an unusually low number of successes if \( P(\text{at most } x) \) is very small (such as 0.05 or less).

EXAMPLE

Consider the procedure of rolling a pair of dice five times and letting \( x \) be the number of times that "7" occurs. The table below describes the probability distribution.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( P(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.402</td>
</tr>
<tr>
<td>1</td>
<td>0.402</td>
</tr>
<tr>
<td>2</td>
<td>( ? )</td>
</tr>
<tr>
<td>3</td>
<td>0.032</td>
</tr>
<tr>
<td>4</td>
<td>0.003</td>
</tr>
<tr>
<td>5</td>
<td>0.000+</td>
</tr>
</tbody>
</table>

(a) Find the value of the missing probability.
(b) Would it be unusual to roll a pair of dice and get at least three "7s"?

EXPECTED VALUE

The expected value of a discrete random variable \( x \) is denoted by \( E \), and it is the mean value of the outcomes, so \( E = \mu \) and \( E \) can also be found by evaluating \( \sum [x \cdot P(x)] \).

That is,

\[
E = \mu = \sum [x \cdot P(x)]
\]
EXAMPLE
When you give the Venetian casino in Las Vegas $5 for a bet on the number 7 in roulette, you have $37/38 probability of losing $5 and you have a $1/38 probability of making a net gain of $175. (The price in $180, including your $5 bet, so the net gain is $175.) If you bet $5 that the outcome is an odd number the probability of losing $5 is $20/38 and probability of making a net gain of $5 is $18/38. (If you bet $5 on an odd number and win, you are given $10 that included your bet, so the net gain is $5.)
(a) If you bet $5 on the number 7, what is your expected value?
(b) If you bet $5 that the outcome is an odd number, what is your expected value?
(c) Which of these options is best: bet on 7, bet on an odd number, or don’t bet? Why?