Section 5.3

Related Hamiltonian-like Properties

TRACEABLE AND HAMILTONIAN CONNECTED GRAPHS

- A graph is traceable if it contains a hamiltonian path.
- A graph $G$ is called an homogeneously traceable if there is a hamiltonian path beginning at every vertex of $G$.
- A graph $G$ is called hypohamiltonian if $G$ is not hamiltonian but $G - v$ is hamiltonian for every vertex $v$.
- We say that $G$ is hamiltonian connected if every two vertices of $G$ are joined by a hamiltonian path.

$(p + 1)$-CLOSURE

For a $(p,q)$ graph $G$, let the $(p + 1)$-closure, denoted by $CL_{p+1}(G)$, be the graph obtained from $G$ by recursively joining pairs of nonadjacent vertices whose degree sum is at least $p + 1$.
A THEOREM ON HAMILTONIAN CONNECTEDNESS

**Theorem 5.3.1 (Bondy and Chvátal):** Let $G$ be a graph of order $p$. If $CL_{p+1}(G)$ is complete, then $G$ is hamiltonian connected.

TWO COROLLARIES

**Corollary 5.3.1:** If $G$ is a graph of order $p$ such that for every pair of distinct nonadjacent vertices $x$ and $y$ in $G$, $\deg x + \deg y \geq p + 1$, then $G$ is hamiltonian connected.

**Corollary 5.3.2:** If $G$ is a graph of order $p$ such that, $\deg x \geq \frac{p+1}{2}$, then $G$ is hamiltonian connected.

PANCONNECTED

A connected graph $G = (V, E)$ is said to be **panconnected** if for each pair of distinct vertices $x$ and $y$, there exists and $x - y$ path of length $l$, for each $l$ satisfying $d(x, y) \leq l \leq |V| - 1$.

**Theorem 5.3.2:** If $G$ is a graph of order $p \geq 4$ such that for every vertex $x \in V(G)$, $\deg x \geq \frac{p+2}{2}$, then $G$ is panconnected.
**PANCYCLIC**

A graph $G$ of order $p$ is **pancyclic** if it contains a cycle of every length $l$, $(3 \leq l \leq p)$. $G$ is **vertex pancyclic** if each vertex of $G$ lies on a cycle of each length $l$, $(3 \leq l \leq p)$.

**Theorem 5.3.3:** If $G$ is a hamiltonian $(p, q)$ graph with $q \geq \frac{p^2}{4}$, then either $G$ is pancyclic or $p$ is even and $G$ is isomorphic to $K_{p/2, p/2}$. 

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