Section 4.1

Networks

A network $N$ is a 5-tuple $N = (V, E, s, t, c)$ where $(V, E)$ is a digraph with distinguished vertices $s$ and $t$ called the source and sink, respectively. Each arc $e$ is assigned a nonnegative real number $c(e)$, called the capacity of the arc; that is, $c$ is a function which assigns nonnegative real numbers to the arcs of $N$.

Note: $\text{in}(v)$ and $\text{out}(v)$ denote the sets of arcs entering and leaving $v$, respectively.

Flow

A (legal) flow $f$ in $N$ is a mapping from the arc set $E$ to the real numbers such that:

1. (Capacity constraint) $f(e) \leq c(e)$ for every $e \in E$

2. (Flow constraint) for each vertex $v$ other than $s$ or $t$,

$$0 = \sum_{e \in \text{in}(v)} f(e) - \sum_{e \in \text{out}(v)} f(e)$$
**LOOPS AND MULTIPLE ARCS**

- Loops never add to the flow because what flows out of the vertex immediately flows back into the same vertex.
- Parallel arcs add nothing since we can replace parallel arcs with a single arc whose flow (and capacity) is the sum of flows (and capacities) on all parallel arcs.
- These restrictions yield $|E| \leq |V|(|V| - 1)$.

**TOTAL FLOW**

Given a network $N$ with flow $f$, the total flow (sometimes called the value of the flow) is defined as:

$$F = \sum_{e \in \text{in}(t)} f(e) - \sum_{e \in \text{out}(t)} f(e)$$

That is, $F$ is the net flow into the sink $t$. We want to maximize $F$. 