Section 1.6
Fundamental Counting

THE PIGEON HOLE PRINCIPLE

Theorem 1.6.1 (The Pigeon Hole Principle): If \( m \) pigeons are placed in \( k \) pigeon holes, then one hole will contain at least \( \left\lceil \frac{m}{k} \right\rceil \) pigeons.

GRAPHS ON SIX VERTICES

Theorem 1.6.2: Any graph on six vertices contains an induced \( K_3 \) or an induced \( \overline{K}_3 \) as a subgraph.
NUMBER OF LABELED GRAPHS ON $p$ VERTICES

**Theorem 1.6.3:** If $N = \binom{p}{2}$ then there are $2^N$ labeled graphs on $p$ vertices.

NUMBER OF SUBGRAPHS OF $K_n$ ISOMORPHIC TO $P_k$

**Theorem 1.6.4:** The number of subgraphs of $K_n$ isomorphic to $P_k$ is

$$\frac{n!}{2(n-k)!}.$$