**Section 11.8: Power Series**

**Introduction:**

So far we have only talked about series of constants. For example,

We asked if such series converge or diverge.

Now, we want to discuss series of *functions*. For example,

Now the question we ask is:

*For what values of does the series converge?*

Another related question is:

*If the series converges for some values of , what function does it converge to; that is, what is ?*

We are only going to discuss very special series of functions called *power series*.

**Power Series and Convergence:**

A series of the form

is called a *power series* centered at . Each partial sum is a polynomial.

Example:

The *coefficients* are:

The *center* is: .

The infinite power series is a function of defined for those values of for which the series converges. The set of values of for which the power series converges is called the *interval of convergence* or the *convergence set*.

The interval of convergence, , has a *radius of convergence*, .

For the power series,

we use the Ratio Test to determine the domain of and thus determine the convergence set.

Notice the Ratio Test tells us that

must be less than 1 for the series to converge. Let

and consider the following three cases:

**CASE 1 ():** If , then the power series is convergence for all since

The interval of convergence is or and the radius of convergence is .

Example:

center:

radius of convergence:

convergence set:

**CASE 2 ():** If , then the power series converges for only since by the Ratio Test the series diverges for all values of except ().

Example:

for all values of except

center:

radius of convergence:

convergence set:

**CASE 3 ():** If and , then by the Ratio Test,

must be less than 1 for the series to converge. The series, not counting endpoints, *converges*

*absolutely* for those values of such that

Thus, is the radius of convergence.

To determine whether the endpoints are included in the domain (interval of convergence), a test *other than the Ratio Test* must be used. (Recall when

the Ratio Test fails.)

Example:

Thus, the series converges when

Hence,

center:

radius of convergence:

Now, before stating the interval of convergence, we need to check the endpoints of the interval, namely and .

When , the power series is

This series diverges by the *n*thTerm Test since

When , the power series is

which also diverges by the *n*thTerm Test since

Thus, the interval of convergence is

the radius of convergence is

and the center is

**Additional Examples:** Find the interval and radius of convergence of the following power series.