**Section 11.6: Absolute Convergence and**

**the Ratio and Root Tests**

**Absolute and Conditional Convergence:**

**Absolute Convergence Test (ACT):** If converges, then also converges.

**Definitions:**

1. is ***absolutely convergent*** or ***converges absolutely*** (CA) if converges.

2. is ***conditionally convergent*** or ***converges conditionally*** (CC) if converges but diverges.

NOTE: After determining convergence by the Alternating Series Test (AST), then use Integral Test, Comparison Test (CT), or Limit Comparison Test (LCT) on to determine absolute convergence (CA) or conditional convergence (CC).

Examples:

**The Ratio Test:**

**The Ratio Test (RT):** Let be a series of nonzero terms and suppose

(i) If , the series converges absolutely.

(ii) If , the series diverges.

(iii) If , the test is inconclusive.

NOTES:

1. For any type of series: positive, alternating, or other.

2. If , the test fails. You *must* use a different test.

3. If , the series diverges ( *ρ* does not have to be finite for this test).

4. If , the series converges ( *ρ* can have a value of 0 in this test).

5. This test is most useful with series involving powers and factorials.

Useful Facts for Factorials:

Examples:

**The Root Test:**

**The Root Test (RoT):** Let be a series of nonzero terms and suppose

(i) If , the series converges absolutely.

(ii) If , the series diverges.

(iii) If , the test is inconclusive.

NOTES:

1. For any type of series: positive, alternating, or other.

2. If , the test fails. You *must* use a different test.

3. If , the series diverges ( *ρ* does not have to be finite for this test).

4. If , the series converges ( *ρ* can have a value of 0 in this test).

5. This test is most useful for series involving powers only.

6. For series involving both powers and factorials use the Ratio Test (RT).

Example: Use the Root Test to determine if the following series diverges or converges.

**Section 11.7: Strategy for Testing Series**

1. If , conclude from the th Term Test that the series diverges.

2. If involves , or , try the Ratio Test.

3. If involves , try the Root Test (or possibly the Ratio Test).

4. If the series is alternating, then obviously try the Alternating Series Test. (Don’t forget to determine absolute or conditional convergence.)

5. If is a positive series and involves only constant powers of , try the Limit Comparison Test. In particular, if is a rational expression in , use this test with as the quotient of the leading terms from numerator and denominator.

6. If the tests above do not work and the series is positive, try the Comparison Test or the Integral Test.

7. If all else fails, try some clever manipulation or a neat “trick” to determine convergence or divergence.

**Summary of Tests for Convergence/Divergence of Series**

|  |  |  |
| --- | --- | --- |
| **Test** | **Convergence** | **Divergence** |
| *n*th-Term Test | , may converge or diverge. Use another test. |  |
| Geometric Series |  |  |
| -Series |  |  |
| Integral Test  (The function must be decreasing to apply this test.) | See improper integrals. | |
| Ratio/Root Test  (Inconclusive if .) | , absolute convergence |  |
| Limit Comparison Test  (The convergence or divergence of is known.) | If (but not infinite), then and either both converge or both diverge. | |
| Comparison Test  (The convergence or divergence of is known.) | converges if converges and . | diverges if diverges and . |
| Alternating Series Test | i. (is decreasing) and  ii. |  |
| Improper Integrals | The limit is a finite number. | The limit either does not exist or is infinite. |