**Section 11.5: Alternating Series**

**Alternating Series:**

**Definition:** An ***alternating series*** is a series whose terms alternate signs. For example,

$$\sum\_{n=1}^{\infty }\left(-1\right)^{n}a\_{n} or \sum\_{n=1}^{\infty }\left(-1\right)^{n-1}a\_{n}$$

**Alternating Series Test (AST):** If the alternating series

$$a\_{1}-a\_{2}+a\_{3}-a\_{4}+…$$

or

$$-a\_{1}+a\_{2}-a\_{3}+a\_{4}-…$$

satisfies

 (i) $a\_{n+1}\leq a\_{n}$ for all $n$; that is, $\left\{a\_{n}\right\}$ is a decreasing sequence, and

 (ii) $\lim\_{n\to \infty }a\_{n}=0$

then the series is convergent.

NOTES:

 (a) The terms of $\left\{a\_{n}\right\}$ must be decreasing ($a\_{n+1}\leq a\_{n}$) and $\lim\_{n\to \infty }a\_{n}=0$. If $\lim\_{n\to \infty }a\_{n}\ne 0$, then the series diverges by the *n*th-Term Test.

 (b) If the series is not an alternating series, $\lim\_{n\to \infty }a\_{n}=0$ does *not* insure convergence!

Examples:

$$1. \sum\_{n=1}^{\infty }\left(-1\right)^{n}\frac{1}{n}$$

$$2. \sum\_{n=1}^{\infty }\left(-1\right)^{n}\frac{1}{n!}$$

$$3. \sum\_{n=1}^{\infty }\frac{\left(-1\right)^{n}n}{2n-1}$$

**Approximating the Sum of an Alternating Series**

**Theorem:** If a convergent alternating series satisfies the condition $a\_{n+1}\leq a\_{n}$, then the absolute value of the remainder $R\_{N}$ involved in approximating the sum $s$ by $s\_{N}$ is less than (or equal to) the first neglected term. That is,

$$\left|s-s\_{N}\right|=\left|R\_{N}\right|\leq a\_{N+1} .$$

Example: Approximate the sum of the following series by its first six terms. Find a bound for the error in your approximation.

$$\sum\_{n=1}^{\infty }\left(-1\right)^{n+1}\frac{1}{n!}$$