**Section 11.2: Series**

**Definitions:**

Let be a sequence of real numbers, then

is the ***infinite series*** (or just the ***series***) associated with the sequence.

Its ***partial sums*** are:

If exists and is finite, then *s* is the ***sum*** of the infinite series and

If *s* exists and is finite, the series ***converges***, otherwise the series ***diverges***.

NOTES:

1. A *sequence* is a listing of numbers, ; a *series* is a sum of numbers,

.

2. Every series involves *two* sequences:

(a) a sequences of terms, , and

(b) a sequence of partial sums, .

Example: Let be a *sequence* where . The *infinite series*,

has *partial sums*:

(A later subsection will show how the formula for was determined.)

In summary,

**Graphing the Partial Sums of a Series on the TI-83/84:**

1. Press **Y=.**

2. In one of the sequence variables, enter the following.

3. Press **2nd, STAT**. Arrow over to **MATH**. Select **5:sum(**.

4. Press **2nd, STAT**. Arrow over to **OPS**. Select **5:seq(**.

5. Enter the formula for the sequence of terms; that is, the *an*.

6. Press the comma (**,**), ***n***, the starting index, ***n***.

7. Finish by closing both sets of parentheses.

Examples: Graph both the sequence of terms and the sequence of partial sums for the following series. Decide based upon your graphs of the partial sums if you think the series converges or diverges.

**Geometric Series:**

is called a ***geometric series*** with ratio *r*.

Let’s determine when a geometric series converges. We do this by considering and .

First, we note that if , which grows without bound, and so diverges. Now, for , we subtract the second equation above from the first and get

If , we know from the last section that and thus

If or , the sequence diverges, and consequently so does .

So, we have the following theorem.

**Theorem:** A geometric series

converges to if and diverges if .

Examples: Determine whether the following series converge or diverge. If the series converges, find its sum.

**Telescoping Series:**

A ***telescoping series***is one in which each partial sum collapses (or telescopes). Sometimes, telescoping series are also called ***collapsing* *series***. See the example below.

Example: Show the following series converges and find its sum.

**A Test for Divergence:**

**The *n*th-Term Test:** If , then diverges.

Examples: Use the *n*th-Term Test to show the following series diverge.

NOTES:

1. If , the series may converge or diverges. (See the next subsection The Harmonic Series.)

2. One can *only conclude divergence* with the *n*th-Term Test!

3. It is necessary that for to converge, but it is not sufficient to conclude convergence!

Proof of the *n*th-Term Test:

We will actually prove what is known as the contrapositive of the *n*th-Term Test. The contrapositive of the *n*th Term Test is:

If converges, then . (1)

Logically, the statement above is equivalent to the original statement of the *n*th­-Term Test.

Now, we proceed to prove statement (1) above. We assume that converges; that is,

Since, , and , then:

Therefore, . By the contrapositive, we have shown that:

If , then diverges.

**The Harmonic Series:**

The series

is called the *harmonic series*.

We note that

However, the harmonic series diverges as we will now show.

What we will show is that the partial sums *­* of the harmonic series grow without bound, that is approaches infinity. Suppose that is large. The *n*th partial sum is

By taking large enough, we can introduce as many ½’s into the last line as we wish. Thus, we see that can be made larger than any number we want; that is, increases without bound (approaches infinity). Hence, diverges which tells us the harmonic series diverges.**Theorem:** If and are convergent series, then so are the series (where *c* is a constant), , and , and

Example: Find the sum of the series

**Theorem:** If diverges and , then also diverges.

Example: Show that the series below diverges.