**Section 11.10: Taylor and Maclaurin Series**

**The Uniqueness Theorem:** Suppose *f* satisfies

for all in some interval around. Then

Thus, a function cannot have more than one power series in that represents it.

NOTES:

1. Consider and the following derivatives:

In each case, if , we have

|  |  |  |
| --- | --- | --- |
|  | Solving for each coefficient: |  |

2. A power series

where coefficients are given by

is called a *Taylor series*.

3. When , the series in known as a *Maclaurin series*.

4. The last part of the Uniqueness Theorem tells us that there is only one power series representation for a function. In particular, the geometric series and differentiation/integration techniques of the last section yield Taylor and Maclaurin series.

Examples:

1. Find the Maclaurin series for .

|  |  |
| --- | --- |
|  |  |

So,

Now, we use the Ratio Test to find the domain.

Note that

Thus, which implies that the radius of convergence is Hence, the domain is all real numbers. So,

2. Find the Maclaurin series for .

Now, we could use the same process as we did in Example 1. However, it is easier if we recognize that . Thus,

Since the domain for is all real numbers, the domain for is also all real numbers; *i.e.*, or .

3. Find the Maclaurin series for .

Let so that .

|  |  |
| --- | --- |
|  |  |

Thus,

and

So,

4. Find the Taylor series for centered at .

Method 1: (Geometric Series)

So,

Hence, . Substituting 1 for , we find that .

Thus,

Verify the convergence and divergence at the endpoints.

Method 2: (Taylor’s Theorem)

|  |  |
| --- | --- |
|  |  |

Hence,

Verify the domain by using the Ratio Test.

5. Find the Maclaurin series for

Method 1: (Geometric Series)

So,

Since this series came from a geometric series, the domain (interval of convergence) is:

or

or

or

Method 2: (Taylor’s Theorem)

Let . Then, .

|  |  |
| --- | --- |
|  |  |

So,

Thus,

Verify the domain by the Ratio Test!

6. Find the Maclaurin series for , where is any real number.

Using Taylor’s Theorem, we have

For *x* = 0, we find that

Therefore, the Maclaurin series for is

This series is called the **binomial series**. Using the Ratio Test, this series will converge if and diverge if . (Verify this!) Convergence at the endpoints, , depends on the value for .

The traditional notation for the coefficients of the binomial series is

and these numbers are called the **binomial coefficients**.

**THE BINOMIAL SERIES:** If is any real number and , then

The series diverges at the endpoints, , if . The series diverges at the endpoint and converges at the endpoint 1 if . The series converges at both endpoints, , if .

7. Represent as a Maclaurin series.

Using the Binomial Series above, we find that

**Please note the table of important Maclaurin series on page 808 of the text.**

**Exercises:**

1. Find the Maclaurin series for . Use the power series for that we developed in Example 2 on page 3. State the domain.

2. Find the Maclaurin series for . Use this result to find the Maclaurin series for . State the domain.

3. Find the Taylor series for about . State the domain.

4. Find the Maclaurin series for . Use the power series for that we developed in Example 1 on page 2. State the domain.

**Answers:**