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10) Degree 4 : Zeros: $1, 2, 2+i$
$\Longrightarrow$ The conjugate is also a root.
$\Longrightarrow$ $2-i$ is a root

15) Degree 6: zeros: $2, 2+i, -3-i, 0$
The conjugates are also zeros or roots.
$\Longrightarrow$ $2-i, -3+i$ are also zeros.

20) Degree 6: zeros: $i, 4-i, 2+i$
Conjugates are also roots: $-i, 4+i, 2-i$

$$P(x) = (x-R_1)(x-R_2)(x-R_3)(x-R_4)$$
$$= (x-i)(x+i)\left(x-(4-i)\right)\left(x-(4+i)\right)\left(x-(2+i)\right)\left(x-(2-i)\right)$$
$$= (x^2+1)\left((x-4)+i\right)\left((x-4)-i\right)\left((x-2)-i\right)\left((x-2)+i\right)$$

$$= (x^2+1)\left((x-4)^2+1\right)\left((x-2)^2+1\right)$$
$$= (x^2+1)(x^2-8x+16+1)(x^2-4x+4+1)$$
$$= (x^2+1)(x^2-8x+17)(x^2-4x+5)$$
$$= (x^4-8x^3+17x^2)(x^2-4x+5) + (x^2-8x+17)(x^2-4x+5)$$
$$= x^6-4x^5+5x^4-8x^5+32x^4-40x^3+17x^4-68x^3+85x^2$$
$$+ x^4-4x^3+5x^2-8x^3+32x^2-40x+17x^2-68x+85$$

$$P(x) = x^6-12x^5+55x^4-120x^3+139x^2-108x+85 \quad ; \; a=1$$

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25) $f(x) = 2x^4 + 5x^3 + 5x^2 + 20x - 12$, zero: $-2i$

$f(x) = (x-2i)(x+2i)\,Q(x)$

$(x^2+4)\,Q(x)$

$\Rightarrow Q(x) = \dfrac{2x^4+5x^3+5x^2+20x-12}{x^2+4}$

$$\begin{array}{r l}
 & 2x^2 + 5x - 3 \\
x^2+4 \,\big)\, & 2x^4 + 5x^3 + 5x^2 + 20x - 12 \\
 & 2x^4 + 8x^2 \\
\hline
 & 5x^3 - 3x^2 + 20x - 12 \\
 & 5x^3 + 20x \\
\hline
 & -3x^2 - 12 \\
 & -3x^2 - 12 \\
\hline
 & 0
\end{array}$$

$\Rightarrow f(x) = (x^2+4)(2x^2+5x-3)$

$= (x^2+4)(2x-1)(x+3)$

$f(x) = 0 \;\rightarrow\; (x^2+4)(2x-1)(x+3) = 0$

$x^2+4=0$	$2x-1=0$	$x+3=0$
$x = \pm 2i$	$x = 1/2$	$x = -3$

Sol. Set $x = \{\pm 2i, 1/2, -3\}$

remaining zeros are $\{2i, 1/2, -3\}$

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30) $g(x) = 2x^5 - 3x^4 - 5x^3 - 15x^2 - 207x + 108$, zero $3i$

$\Rightarrow -3i$ is also a root.

$g(x) = (x-3i)(x+3i) \cdot Q(x)$

$= (x^2+9) \cdot Q(x)$

$$Q(x) = \frac{g(x)}{(x^2+9)}$$

$$\begin{array}{r|l} & 2x^3 - 3x^2 - 23x + 12 \\ \hline x^2+9 & 2x^5 - 3x^4 - 5x^3 - 15x^2 - 207x + 108 \\ & 2x^5 \quad + 18x^3 \\ \hline & -3x^4 - 23x^3 - 15x^2 \\ & -3x^4 \quad - 27x^2 \\ \hline & -23x^3 + 12x^2 - 207x \\ & -23x^3 \quad - 207x \\ \hline & 12x^2 + 108 \\ & 12x^2 + 108 \\ \hline & 0 \quad 0 \end{array}$$

$g(x) = (x^2+9)(2x^3 - 3x^2 - 23x + 12)$

$g(1/2) = 0 \Rightarrow x = 1/2$ is a root $\Rightarrow (x - 1/2)$ is a factor

$\times$	2	-3	-23	12
$1/2$	$\times$	1	-1	-12
$\times$	2	-2	-24	0

$g(x) = (x^2+9)(x - 1/2)(2x^2 - 2x - 24)$

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Continue #30)

$$g(x) = (x^2+9)(x-1/2)2(x^2-x-12)$$

$$= 2(x^2+9)(x-1/2)(x-4)(x+3)$$

$\Rightarrow$ Remaining zeros are: $\{x = -3i, 1/2, 4, -3\}$

#35) $f(x) = x^4+5x^2+4$

$$= (x^2+4)(x^2+1)$$

$f(x)=0 \implies (x^2+4)(x^2+1) = 0$

$x^2+4=0$	$x^2+1=0$
$x^2=-4$	$x^2=-1$
$x=\pm 2i$	$x=\pm\sqrt{-1}=\pm i$

$\therefore$ Roots Are $= \{\pm 2i, \pm i\}$; $f(x) = (x-i)(x+i)(x-2i)(x+2i)$

40) $f(x) = 2x^4+x^3-35x^2-113x+65$

List of Rational zeros $= \frac{p}{q}$

p = factors of $65 = \pm 1, \pm 5, \pm 13, \pm 65$

q = factors of $2 = \pm 1, \pm 2$

List of Rational zeros $= \pm 1, \pm 1/2, \pm 5, \pm 5/2, \pm 13, \pm 13/2$

$\pm 65, \pm 65/2$.

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Continue #40)

$f(1/2) = 0 \Rightarrow x = 1/2$ is a root

$\Rightarrow (x - 1/2)$ is a factor.

$f(5) = 0 \Rightarrow x = 5$ is a root

$\Rightarrow (x-5)$ is a factor.

$\rightarrow f(x) = (x - 1/2)(x-5)\ Q(x)$

$= (x^2 - 11/2\, x + 5/2)\ Q(x)$

$$Q(x) = \frac{f(x)}{x^2 - 11/2\, x + 5/2} = \frac{f(x)}{1/2\,(2x^2 - 11x + 5)} = 2\frac{f(x)}{(2x^2-11x+5)}$$

$$\begin{array}{r|l} & 2x^2 + 6x + 13 \\ 2x^2-11x+5 & 2x^4 + x^3 - 39x^2 - 113x + 65 \\ & \underline{2x^4 - 11x^3 + 5x^2} \\ & 12x^3 - 40x^2 - 113x + 65 \\ & \underline{12x^3 - 66x^2 + 30x} \\ & 26x^2 - 143x + 65 \\ & \underline{26x^2 - 143x + 65} \\ & 0 \end{array}$$

$\therefore f(x) = (x - 1/2)(x-5)\,2(x^2+6x+13) = 2(x-1/2)(x-5)(x^2+6x+13)$

To find the zeros of $f(x)$, set $f(x) = 0$.

$\Rightarrow 2(x - 1/2)(x-5)(x^2+6x+13) = 0$

$x - 1/2 = 0$	$x - 5 = 0$	$x^2+6x+13 = 0$
$x = 1/2$	$x = 5$	

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Continue # 4c)

Again we have:

$$2\left(x-\tfrac{1}{2}\right)(x-5)\left(x^2+6x+13\right)=0$$

$x-\frac{1}{2}=0$	$x-5=0$	$x^2+6x+13=0$
$x=\frac{1}{2}$	$x=5$	

$x^2+6x+13=0$

$a=1,\ b=6,\ c=13$

$b^2-4ac = 36-4\cdot1\cdot13 = 36-52$

$= -16 < 0$

$\Rightarrow x = \dfrac{-6\pm\sqrt{-16}}{2\times1} = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \dfrac{-6\pm4i}{2} = \dfrac{2(-3\pm2i)}{2}$

$= -3\pm2i$

So the complex zeros are: $\frac{1}{2}, 5, -3-2i, -3+2i$

$f(x) = 2\left(x-\frac{1}{2}\right)(x-5)\left(x-(-3-2i)\right)\left(x-(-3+2i)\right)$

$f(x) = 2\left(x-\frac{1}{2}\right)(x-5)(x+3+2i)(x+3-2i)$