Study Guide for MATH 1111 F.E.

Dr. ZABIDAWI

1) Solve $\sqrt{2x+15} = x+6$

$\Rightarrow 2x+15 = (x+6)^2 = x^2+12x+36$

$\Rightarrow x^2+10x+21 = 0$

$(x+7)(x+3) = 0$

$x+7=0$	$(x+3)=0$
$x=-7$	$x=-3$

Check $x=-7$

$\sqrt{2(-7)+15} \stackrel{?}{=} -7+6$

$1 \neq -1$ ✗

Check $x=-3$

$\sqrt{2(-3)+15} \stackrel{?}{=} -3+6$

$\sqrt{9} = 3$ ✓

∴ Sol. Set $x = \{-3\}$.

2) $|3x-7| = 4$

$3x-7=-4$	$3x-7=4$
$3x=7-4=3$	$3x=7+4=11$
$x=\frac{3}{3}=1$	$x=\frac{11}{3}$

Sol. Set $x = \{1, 11/3\}$

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Solve

3) $x^2+4x \geq -3$

$x^2+4x+3 \geq 0$

Let $f(x) = x^2+4x+3$

$= (x+3)(x+1)$

Critical values are the x-values that make $f(x)=0$

$\Rightarrow$ $x=-3, -1$ are critical values.

x	$-\infty$		-3		-1		∞
$f(x)$		$+$	ϕ	$-$	ϕ	$+$	

Sol. Set $x \in (-\infty, -3] \cup [-1, \infty)$

4) Solve: $2 + |4x-6| < 12$

$|4x-6| < 12-2$

$|4x-6| < 10$

$-10 < 4x-6 < 10$

$-4 < 4x < 16$

$-1 < x < 4$ $\Rightarrow$ Sol. Set $x \in (-1,4)$

$x \in (-1,4)$.

5) Solve $\frac{3x}{x+2} \leq \frac{1}{x+2}$

$\Rightarrow$ $\frac{3x-1}{x+2} \leq 0$

Let $f(x) = \frac{3x-1}{x+2}$

Critical values are the x-values that make $f(x) = 0, \pm\infty$

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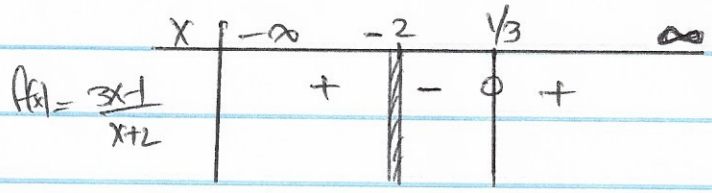
#5) $\frac{3x-1}{x+2} \leq 0$

Critical Value : $3x-1=0 \Rightarrow x = 1/3$

$x+2=0 \Rightarrow x=-2$



Sol Set $x \in (-2, 1/3]$.

#6) Determine Symmetry

a) $y^2 = -x+1$ Symmetric w.r.t. x-axis

b) $x^2+y-16=0$ Symmetric w.r.t y-axis.

c) $4x^2+y^2=4$ Symmetric w.r.t. x-axis, y-axis, origin.

d) $y = \frac{4x}{x^2+16}$ Symmetric w.r.t origin.

e) $y = x^3+x^2+6x+9$ No Symmetry.

#7) $3x-6y=12$

x- Intercept $\rightarrow y=0 \Rightarrow 3x=12$; $x=4$; $(4,0)$

y- Intercept $\Rightarrow x=0 \rightarrow -6y=12$, $y=-2$ $(0,-2)$

#8) Find eq. of the St. line containing $(1,-1)$ and $(2,3)$

$$m = \frac{3+1}{2-1} = \frac{4}{1} = 4$$

$(y+1) = 4(x-1)$

$\rightarrow y = 4x-4-1 \Rightarrow y = 4x-5$

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#9) Find the Center and radius of a Circle:

$x^2+y^2-6x+8y = 11$

$x^2-6x+y^2+8y = 11$

$x^2-6x+9+y^2+8y+16 = 11+9+16$

$(x-3)^2+(y+4)^2 = 36$

$\therefore$ Circle Center $(3,-4)$, Radius $= 6$

#10) Fin the eq. of the line which is $\perp$ to $x-7y=3$ and passes thru $(5,5)$

$x-7y = 3$

$-7y = -x+3$

$y = \frac{1}{7}x - \frac{3}{7} \longrightarrow m = \frac{1}{7}$

$\Longrightarrow m_{\perp} = -7$

$\Longrightarrow (y-5) = -7(x-5)$

$y = -7x+35+5$

$y = -7x+40$

#11) If $f(x) = 4x^2-2x+3$, find $f(0)-f(3)$

$f(0) = 3$, $f(3) = 4(3)^2-2(3)+3 = 36-6+3 = 33$

$\longrightarrow f(0)-f(3) = 3-33 = -30$

#12) Does $(2,5) \in f(x) = 3x^2-6x+5$

$f(2) = 3(2)^2-6(2)+5 = 12-12+5 = 5$

$\therefore (2,5) \in f(x)$ YES because $f(2)=5$

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#13) Find the domain of $f(x) = \sqrt{5+4x-x^2}$

i) Domain $5+4x-x^2 \geq 0$

$x^2-4x-5 \leq 0$

$(x-5)(x+1) \leq 0$

Let $f(x) = (x-5)(x+1)$

Critical values are the x-values that make $f(x) = 0$.

$\Rightarrow$ $x = 5, -1$ are Critical values.

x	$-\infty$		-1		5		∞
$f(x) = (x-5)(x+1)$		+	0	−	0	+	

ii. Sol. Set $x \in [-1, 5]$ = Domain.

#14) a) $g(x) = 4x^3 - x^4 - x^2 + \sqrt{5x}$; Not a polynomial because $x^{1/2}$

b) $g(x) = 4x^7 - \pi x^4 + \frac{3}{2}x^2 + \sqrt{5}x - 6$; Polynomial, degree = 7

c) $g(x) = 3x^4 + 3x^2 + 1$; Polynomial of degree 4

d) $g(x) = 4x^3 + x - \frac{1}{x-2}$; Not a polynomial because of $\frac{1}{x-2}$

e) $g(x) = -4x + 1$; Polynomial = Binomial of degree 1

f) $g(x) = 4$; Polynomial = Monomial of degree 0.

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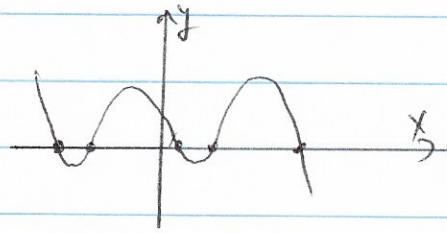
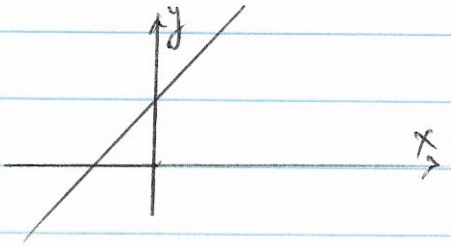
15) Assume that the graphs in the figure below are those of polynomials. What is the least possible degree of each polynomial?



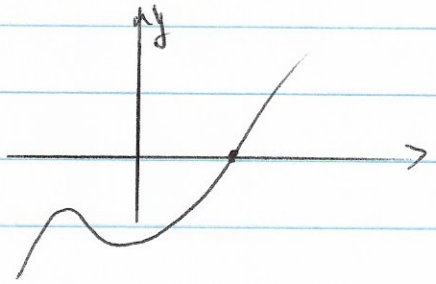
Degree = 5.

b)



Degree = 1

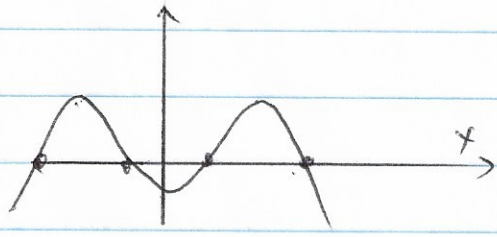
c)



Degree = 3
One Real Root
Two Imaginary Roots.

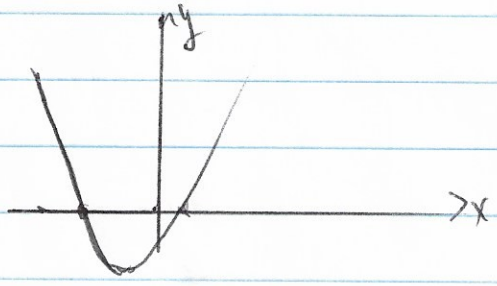
2 turn at Most = n-1 → n = 3

d)



Degree = 4

e)



Degree = 2

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#16) Use the factor theorem to determine if the following are factors of x^3+4x^2+x-6

a) $(x-6)$; $f(6)=360 \longrightarrow$ No ; $(x-6)$ is Not a factor of $f(x)$

b) $(x+1)$; $f(-1)=-4 \Rightarrow$ NO, $(x+1)$ is Not a factor of $f(x)$

c) $(x-1)$, $f(1)=0 \Rightarrow$ Yes. $(x-1)$ is a factor of $f(x)$

d) $(x+3)$, $f(-3)=0 \Rightarrow$ Yes, $(x+3)$ is a factor of $f(x)$

e) $(x-3)$, $f(3)=60 \Rightarrow$ NO, $(x-3)$ is Not a factor of $f(x)$

f) $(x+2)$; $f(-2)=0 \Rightarrow$ YES, $(x+2)$ is a factor of $f(x)$

#17) Factor $f(x)=2x^3+x^2-5x+2$ Completely.

$a_0=2$, $a_n=2$

P = factors of $a_0 = \pm 1, \pm 2$

q = factors of $a_n = \pm 1, \pm 2$

Rat of Rational zero $= \pm 1, \pm \frac{1}{2}, \pm 2$

$f(1) = 2(1)^3+(1)^2-5(1)+2 = 2+1-5+2 = 0.$

$\Rightarrow (x-1)$ is a factor of $f(x)$

1	2	1	−5	2
		2	3	−2
	2	3	−2	0 = Remainder

$f(x) = (x-1)(2x^2+3x-2) = (x-1)(2x-1)(x+2)$

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#18) Solve for x: $x^6 + 7x^3 - 8 = 0$: Quadratic in Form

Let $z = x^3 \Rightarrow z^2 = x^6$

Substitute: $z^2 + 7z - 8 = 0$

$(z+8)(z-1) = 0$

$\Rightarrow z = -8, 1$

Decode $z = x^3$

$z = -8$	$z = 1$
$x^3 = -8$	$x^3 = 1$
$x = -2$	$x = 1$

Sol. Set $x = \{-2, 1\}$

#19) Solve the following system of equations.

$3x + 7y = 15$ — (1)

$2x + 5y = 11$ — (2)

$-2 \times$ eq(1): $-6x - 14y = -30$ — (1)

$3 \times$ eq(2): $6x + 15y = 33$ — (2)

$y = 3$

eq (1) $\Rightarrow 3x = 15 - 7y$

$x = \frac{15-7y}{3} = \frac{15-21}{3} = -2$

$\therefore$ Sol. Set $x = -2, y = 3$

$(-2, 3)$ Point of Intersection.

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#20) Solve the following system of equations.

$$\begin{aligned} x+y+2z &= 1 \quad \text{---- (1)} \\ 3x+2y-z &= 9 \quad \text{---- (2)} \\ 2x-y+z &= 2 \quad \text{---- (3)} \end{aligned}$$

$-3\times$ eq(1) $\quad -3x-3y-6z = -3 \quad$ ---- (1)

eq(2) $\quad 3x+2y-z = 9 \quad$ ---- (2)

$$-y-7z = 6$$

$$y+7z = -6 \quad \text{---- (4)}$$

$-2\times$ eq(1) $\quad -2x-2y-4z = -2 \quad$ ---- (1)

eq(3) $\quad 2x-y+z = 2 \quad$ ---- (2

$$-3y-3z = 0$$

$$y+z = 0 \quad \text{---- (5)}$$

$$y+7z = -6 \quad \text{---- (4)}$$

$$y+z = 0 \quad \text{---- (5)}$$

eq(4) - eq(5): $\quad 6z = -6$

$$\boxed{z = -1}$$

eq(5) $\Rightarrow y = -z \Rightarrow \boxed{y = 1}$

eq(1) $\Rightarrow x = 1-y-2z$

$= 1-1-2(-1) = 2$, $\boxed{x=2}$

$\therefore$ Sol Set $x=2, y=1, z=-1$; $(2,1,-1)$ Point of Intersection

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#21) $PH = -\log [H^+]$ where H^+ = Hydrogen ion concentration in moles per liters.

If $PH = 6.5$ Find H^+

$\Rightarrow 6.5 = -\log [H^+]$

$\log [H^+] = -6.5 \Rightarrow H^+ = 10^{-6.5} = 3.2 \times 10^{-7} \frac{\text{Moles}}{L}$

#22) Simplify the following

a) $\left(\frac{-2r^{-2}st^3}{(4rs^{-1})^2 t^{-2}}\right)^{-1} = \frac{(4rs^{-1})^2 t^{-2}}{-2r^{-2}st^3} = \frac{16 r^2 s^{-2} t^{-2}}{-2r^{-2}s\ t^3}$

$= \frac{-8r^4}{s^3 t^5} = \frac{-8r^4}{s^3 t^5}$

b) $\frac{4x^{-2}(yz)^{-1}}{2^3 x^4 y} = \frac{4}{8\, x^4 y \cdot x^2 (yz)^1} = \frac{1}{2x^6 y^2 z}$

#23) If $27 = 3^{(2x+1)}$

a) $\log_3 27 = (2x+1)$

b) $\log 27 = \log 3^{(2x+1)} = (2x+1)\log 3$

c) $\ln 27 = \ln 3^{(2x+1)} = (2x+1)\ln 3$

d) $27 = 3^{2x+1}$

$3^3 = 3^{2x+1} \longrightarrow 3 = 2x+1$

$\Rightarrow 2x = 3-1 = 2 \Rightarrow \boxed{x=1}$

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#24) Compute fog for the following functions:

a) $f(x) = x^2+1$, $g(x) = 2x^2+3$

$$(fog)(x) = (2x^2+3)^2+1 = 4x^4+12x^2+9+1 = 4x^4+12x^2+10$$

b) $f(x) = \frac{x}{x-1}$; $g(x) = -\frac{4}{x}$

$$(fog)(x) = \frac{-4/x}{-4/x-1} = \frac{-4}{-4-x} = \frac{4}{4+x}$$

#25) Write the following as a single logarithm.

a) $$\log_3(2x^2-x-1) - \log_3(x-1)$$

$$= \log_3\left(\frac{2x^2-x-1}{x-1}\right) = \log_3\frac{(2x+1)(x-1)}{(x-1)} \quad ; x \neq 1$$

$$= \log_3(2x+1) \quad = \log_3(2x+1)$$

b) Use logarithms rules to express the following as the sum and/or difference of logarithms. Write powers as factors.

$$\log_2\left(\frac{\sqrt{z}\,y^2}{x^2}\right)$$

$$= \log_2(\sqrt{z}\,y^2) - \log_2 x^2$$

$$= \log_2(z^{1/2}\cdot y^2) - 2\log_2 x$$

$$= \log_2 z^{1/2} + \log_2 y^2 - 2\log_2 x$$

$$= \frac{1}{2}\log_2 z + 2\log_2 y - 2\log_2 x$$

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#26) Solve: $e^{4x-1} = 24$

$\Rightarrow \ln e^{(4x-1)} = \ln 24$

$(4x-1)\ln e = \ln 24 \quad ; \ \ln e = 1$

$4x-1 = \ln 24$

$4x = \ln 24 + 1$

$x = \frac{(\ln 24 + 1)}{4}$

#27) Solve for x: $4 - \log_3 x = \log_3 (x-24)$

$\Rightarrow 4 = \log_3 x + \log_3 (x-24)$

$4 = \log_3 x(x-24)$

$\Leftrightarrow x(x-24) = 3^4 = 81$

$x^2 - 24x - 81 = 0$

$(x-27)(x+3) = 0$

$\Rightarrow$ $x-27=0$ | $x+3=0$

$x=27$ | $x=-3$ Rejected because $\log_3(+)$

Sol. Set $x=27$

#28) If $1200 is invested for 9 years at an annual percentage Rate APR of 2.3%, determine the amount of money when it is Compounded:

a) Semiannually $\Rightarrow n=2$, $A = 1200\left(1+\frac{.023}{2}\right)^{(2\times 9)} = \$1474.24$

b) monthly $\Rightarrow n=12$, $A = 1200\left(1+\frac{.023}{12}\right)^{(12\times 9)} = \$1475.69$

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#28) c) Daily $\Rightarrow n = 365 \rightarrow A = 1200\left(1+\frac{.023}{365}\right)^{(365\times 9)} = \$1475.97$

d) Continuously $\Rightarrow n \to \infty$, $A = Pe^{RT}$

$= 1200\, e^{(.023\times 9)} = \$1475.98$

#29) Simplify: Use only positive exponents in your solution.

$$\frac{-24x^6y^4(z^2)^3}{-6x^2y^4z^8} = \frac{4x^4z^6}{z^8}$$

$$= \frac{4x^4}{z^2}$$