

ZABI DAWI

MATH 1101

Solution to Problem # 30 in Section 5.4

Find the center and radius of the circle in the xy -plane that passes through the given points $P, Q,$ and $R.$

$Q(0,22), R(17,15)$

Solution:

Eq. of Circle is: $(x-h)^2 + (y-k)^2 = r^2$

where (h,k) is the center

and $r =$ Radius.

$h^2 + (15-k)^2 = r^2$

$(-7+h)^2 + (15-k)^2 = r^2$

$(7+h)^2 + (15-k)^2 = r^2$

$49 + 14h + h^2 + 225 - 30k + k^2 = r^2$ — (1)

$\Rightarrow (0-h)^2 + (22-k)^2 = r^2$

$h^2 + 484 - 44k + k^2 = r^2$ — (2)

$\Rightarrow (17-h)^2 + (15-k)^2 = r^2$

$289 - 34h + h^2 + 225 - 30k + k^2 = r^2$ — (3)

Dn ZABIDAWI

Continue #30 in Section 5.4

$$\text{We have: } h^2 + k^2 + 14h - 30k + 274 = r^2 \quad (1)$$

$$h^2 + k^2 - 44k + 484 = r^2 \quad (2)$$

$$h^2 + k^2 - 34h - 30k + 514 = r^2 \quad (3)$$

$$\text{Eq. (1)} - \text{Eq. (2)} \Rightarrow 14h + 14k = 210 \quad (4)$$

$$\text{Eq. (1)} - \text{Eq. (3)} \Rightarrow 48k = 240$$

$$\Rightarrow k = \frac{240}{48} = 5 ; \boxed{h=5}$$

$$\text{Eq. (4)} \Rightarrow 14k = 210 - 14h$$

$$= 210 - 70 = 140$$

$$\Rightarrow k = \frac{140}{14} = 10 ; \boxed{h=10}$$

\(\therefore\) The Center of the Circle is $(5, 10)$.

$$\text{Eq. (2)} \Rightarrow r^2 = (5)^2 + (10)^2 - 44 \times 10 + 484$$

$$= 25 + 100 - 440 + 484 = 169$$

$$r^2 = 169 \Rightarrow r = 13$$

$$\therefore \boxed{\text{Radius} = 13}$$