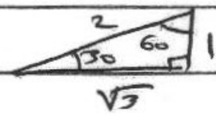
Dr. ZABDAWI

Section 7.4:



10) $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$$= \sin\pi/3 \cos\frac{\pi}{4} - \cos\pi/3 \cos\frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3} \cdot \sqrt{2}}{2 \times 2} - \frac{1 \times \sqrt{2}}{2 \times 2}$$

$$= \frac{\sqrt{2}\left(\sqrt{3} - 1\right)}{4}$$

15) $\tan 15^\circ = \tan\left(45^\circ - 30^\circ\right)$

$$= \frac{\tan 45 - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ} = \left(\frac{1 - \frac{1}{\sqrt{3}}}{1 + 1 \cdot \frac{1}{\sqrt{3}}}\right)\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$= \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$$

20) $\cot\left(-\frac{5\pi}{12}\right) = -\cot\left(\frac{5\pi}{12}\right)$

But $\frac{5\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4} = \frac{2\pi + 3\pi}{12} = \frac{5\pi}{12}$

$\Rightarrow \cot\left(-\frac{5\pi}{12}\right) = -\cot\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$$= -\frac{1}{\tan\left(\pi/6 + \pi/4\right)}$$

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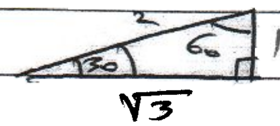
20) $$\cot\left(-\frac{5\pi}{12}\right) = \frac{-1}{\tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right)}$$



$$\tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right) = \frac{\tan\pi/6 + \tan\pi/4}{1-\tan\frac{\pi}{6}\cdot\tan\frac{\pi}{4}}$$

$$= \left(\frac{\frac{1}{\sqrt{3}}+1}{1-\frac{1}{\sqrt{3}}\cdot 1}\right)\left(\frac{\sqrt{3}}{\sqrt{3}}\right)$$

$$= \frac{1+\sqrt{3}}{\sqrt{3}-1}$$

$$\therefore \cot\left(-\frac{5\pi}{12}\right) = \frac{-(\sqrt{3}-1)}{1+\sqrt{3}} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

25) $$\frac{\tan 20^\circ + \tan 25^\circ}{1-\tan 20^\circ\cdot\tan 25^\circ} = \tan(20^\circ+25^\circ)$$

$$= \tan 45^\circ = 1$$

30) $$\sin\frac{\pi}{18}\cos\frac{5\pi}{18} + \cos\frac{\pi}{18}\sin\frac{5\pi}{18}$$

Recall that $\sin\alpha\cos\beta + \cos\alpha\sin\beta = \sin(\alpha+\beta)$

$$\Rightarrow \sin\left(\frac{\pi}{18}\right)\cos\frac{5\pi}{18} + \cos\frac{\pi}{18}\sin\frac{5\pi}{18} = \sin\left(\frac{\pi}{18}+\frac{5\pi}{18}\right)$$

$$= \sin\left(\frac{6\pi}{18}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

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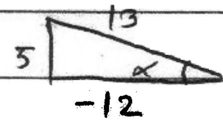
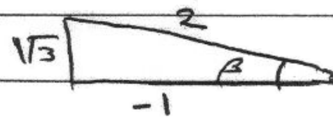
#35) $\sin\alpha = \frac{5}{13}$, $-\frac{3\pi}{2} < \alpha < -\pi$, $\tan\beta = -\sqrt{3}$, $\frac{\pi}{2} < \beta < \pi$

$\Rightarrow$ α is in QII



$\Rightarrow$ β is in QII



a) $$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \frac{5}{13} \times -\frac{1}{2} + \frac{-12}{13} \times \frac{\sqrt{3}}{2}$$
$$= \frac{-5-12\sqrt{3}}{26} = -\frac{(5+12\sqrt{3})}{26}$$

b) $$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= \frac{-12}{13} \times -\frac{1}{2} - \frac{5}{13} \times \frac{\sqrt{3}}{2}$$
$$= \frac{12-5\sqrt{3}}{26}$$

c) $$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$= \frac{5}{13} \times -\frac{1}{2} + \frac{12}{13}\,\frac{\sqrt{3}}{2}$$
$$= \frac{-5+12\sqrt{3}}{26}$$

d) $$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha.\tan\beta} = \left[\frac{\frac{5}{-12} + \frac{\sqrt{3}}{1}}{1+\left(\frac{5}{-12} \times \frac{\sqrt{3}}{-1}\right)}\right]\left[\frac{12}{12}\right]$$
$$= \frac{-5+12\sqrt{3}}{12+5\sqrt{3}}$$

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$$\tan(\alpha-\beta) = \left(\frac{-5+12\sqrt{3}}{12+5\sqrt{3}}\right)\left(\frac{12-5\sqrt{3}}{12-5\sqrt{3}}\right)$$

Recall that $(a+b)(a-b) = a^2-b^2$

$$\Longrightarrow \tan(\alpha-\beta) = \frac{(-5+12\sqrt{3})(12-5\sqrt{3})}{12^2-(5\sqrt{3})^2}$$

$$= \frac{-60+25\sqrt{3}+144\sqrt{3}-60\times 3}{144-25\times 3}$$

$$= \frac{-240+169\sqrt{3}}{69}$$