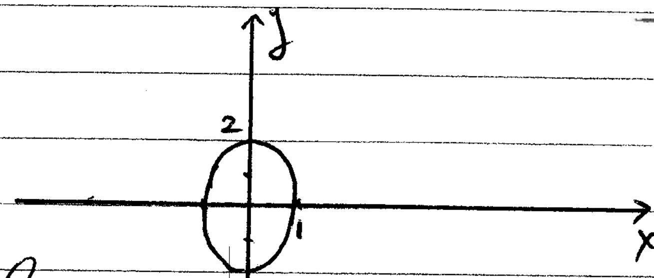
Section 10.3 Dr. ZABDAWI

#15)

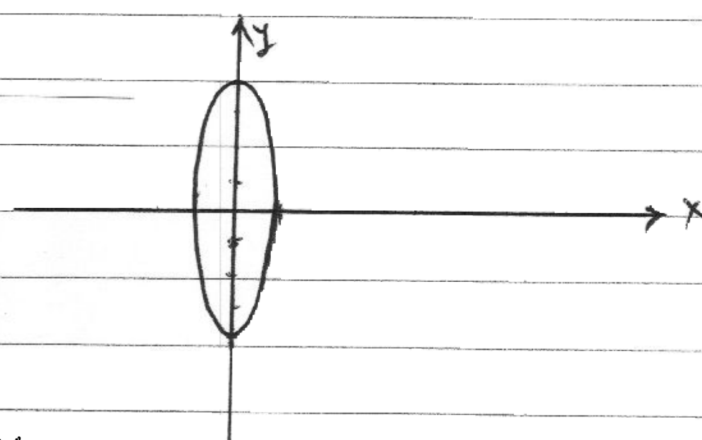


Y-axis is the major axis, $a=2$
x-axis is the minor axis, $b=1$

$\therefore \frac{x^2}{1}+\frac{y^2}{4}=1$ which is B,

#20) $x^2+\frac{y^2}{16}=1$

$a^2=16 \Longrightarrow a=\pm 4$, Majors $(0,-4)$, $(0,4)$
$b^2=1 \Longrightarrow b=\pm 1$, Minors $(-1,0)$, $(1,0)$



#25) $x^2+y^2=16$
$\Longrightarrow \frac{x^2}{16}+\frac{y^2}{16}=1$; This is a special ellipse where $a=b=4$

Major = Minor , the ellipse is a circle with radius $=4$
Center $(0,0)$

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#30) Center $(0,0)$, focus at $(0,1)$; vertex at $(0,-2)$

Focus is on the y-axis $\Rightarrow$ Major axis is the y-axis

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$a = 2$, $c = 1$; $c^2 = a^2 - b^2$

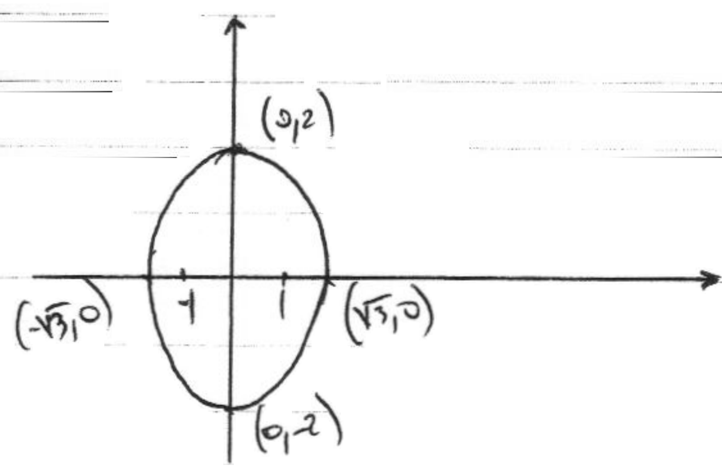
$\Rightarrow b^2 = a^2 - c^2 = 2^2 - 1^2 = 4 - 1 = 3$

$\therefore$ Eq. is $\frac{x^2}{3} + \frac{y^2}{4} = 1$

$a^2 = 4 \Rightarrow a = \pm 2$, Majors $(0,-2)$, $(0,2)$

$b^2 = 3 \Rightarrow b = \pm\sqrt{3}$, Minors $(-\sqrt{3},0)$, $(\sqrt{3},0)$

$c^2 = a^2 - b^2 = 4 - 3 = 1 \Rightarrow c = \pm 1$, Foci $(0,-1)$, $(0,1)$



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#35) Foci at $(0,\pm 3)$, x-Intercepts are ± 2.
Foci are on the y-axis $\Rightarrow$ Y-axis is the major axis.

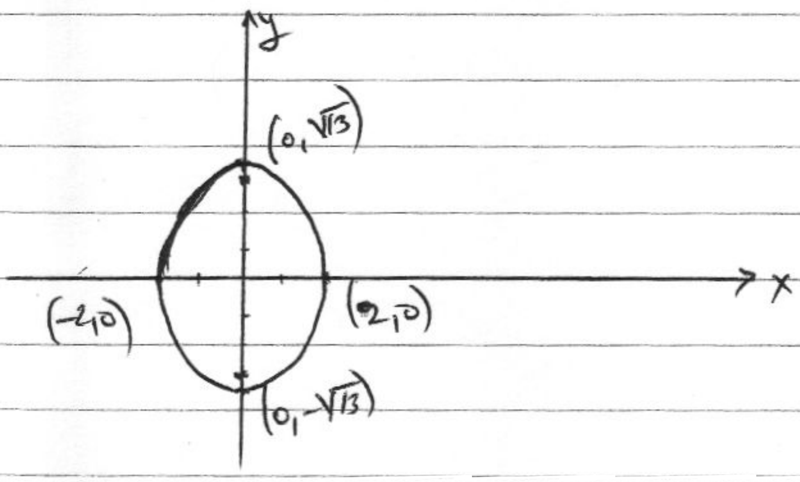
$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

$c=3$, $b=2$, $c^2=a^2-b^2 \Rightarrow a^2=c^2+b^2$
$a^2=9+4=13$

∴ Eq. of the ellipse is:

$$\frac{x^2}{4}+\frac{y^2}{13}=1$$

$a^2=13 \Rightarrow a=\pm\sqrt{13}$; Majors $(0,-\sqrt{13})$, $(0,\sqrt{13})$
$b^2=4 \Rightarrow b=\pm 2$; Minors $(-2,0)$, $(2,0)$
$c^2=a^2-b^2=13-4=9 \Rightarrow c=\pm 3$, Foci $(0,-3)$, $(0,3)$



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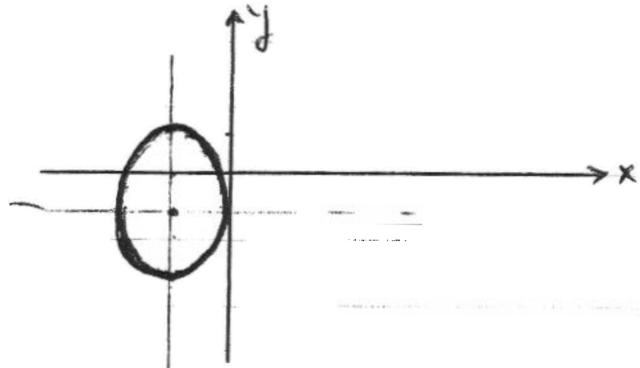
#40) Center $(-1,1)$
Major axis is the Y-axis



$$\rightarrow \frac{(x+1)^2}{b^2} + \frac{(y-1)^2}{a^2} = 1$$

$a =$ Distance from Vertex to Center $= 2$, $a^2 = 4$
$b =$ Distance from Minor to Center $= 1$, $b^2 = 1$

$$\therefore \frac{(x+1)^2}{1} + \frac{(y-1)^2}{4} = 1$$

#45) $(x+5)^2 + 4(y-4)^2 = 16$

$$\rightarrow \frac{(x+5)^2}{16} + \frac{(y-4)^2}{4} = 1$$

Ellipse: Center $(-5,4)$
let $X = x+5$, $Y = y-4$ Notice $x = X-5$, $y = Y+4$
From $(-5,4)$ we have.

$$\frac{X^2}{16} + \frac{Y^2}{4} = 1$$

Ellipse: $a^2 = 16 \Rightarrow a = \pm 4$; Majors $(-4,0)$, $(4,0)$
$b^2 = 4 \Rightarrow b = \pm 2$; Minors $(0,-2)$, $(0,2)$
$c^2 = a^2 - b^2 =$, Foci $(-\sqrt{12},0)$, $(\sqrt{12},0)$

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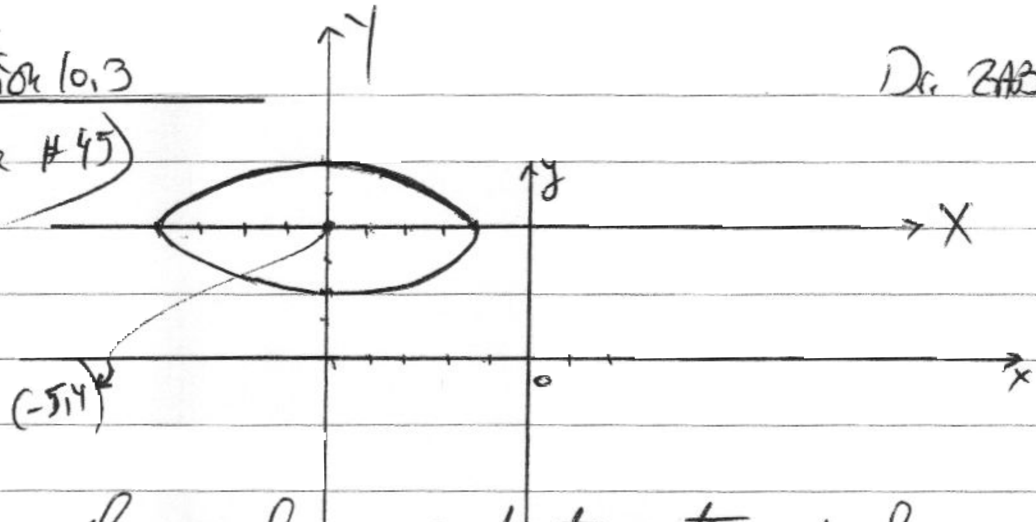
Continue #45)



From the original x-y coordinate system, we have:

Ellipse: Center $(-5,4)$

Simply add the coordinates of the center

$$\begin{cases} \text{Major: } (-9,4),\ (-1,4) \\ \text{Minor } (-5,-2+4),\ (-5,\ 2+4) = (-5,2),(-5,6) \\ \text{Foci } (-5-\sqrt{12},4),\ (-5+\sqrt{12},4) \end{cases}$$

$$4\left(x^2+2x+1-1\right) + 3\left(y^2-2y+1-1\right) = 5$$

$$4\left(x^2+2x+1\right)-4 + 3\left(y^2-2y+1\right)-3 = 5$$

$$4\left(x+1\right)^2 + 3\left(y-1\right)^2 = 5+3+4 = 12$$

$$\frac{4\left(x+1\right)^2}{12} + \frac{3\left(y-1\right)^2}{12} = 1$$

$$\frac{\left(x+1\right)^2}{3} + \frac{\left(y-1\right)^2}{4} = 1$$

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Continue #50

$$\frac{(x+1)^2}{3} \quad \frac{(y-1)^2}{4} = 1$$

Ellipse with Center $(-1,1)$

Now: let $X = x+1$, $Y = y-1$; Notice $x = X-1$, $y = Y+1$

From $(-1,1)$ we have:

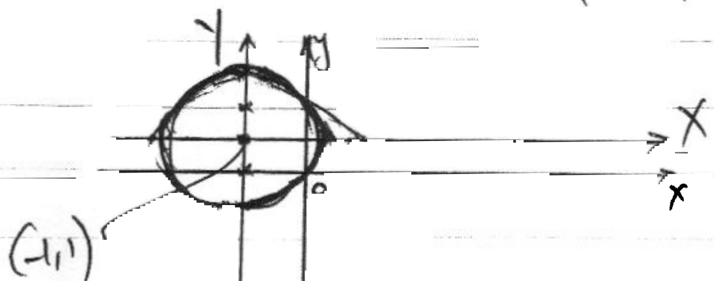
$$\frac{X^2}{3} \quad \frac{Y^2}{4} = 1$$

Ellipse : Center $(0,0)$

$a^2 = 4 \Rightarrow a = \pm 2$, Majors $(0,-2)$, $(0,2)$

$b^2 = 3 \Rightarrow b = \pm\sqrt{3}$, Minors $(-\sqrt{3},0)$, $(\sqrt{3},0)$

$c^2 = a^2 - b^2 = 4-3 = 1 \Rightarrow c = \pm 1$, Foci $(0,-1)$, $(0,1)$.



From the original xy coordinate system we have an Ellipse with:

Center $(-1,1)$

Major $(-1+0, 1-2)$, $(-1+0, 1+2) = (-1,-1)$, $(-1,3)$

Minor $(-1-\sqrt{3}, 1)$, $(-1+\sqrt{3}, 1)$

Foci $(-1,0)$, $(-1,2)$

I am simply adding the coordinates of the Center $(-1,1)$

Recall $x = X-1$

$y = Y+1$

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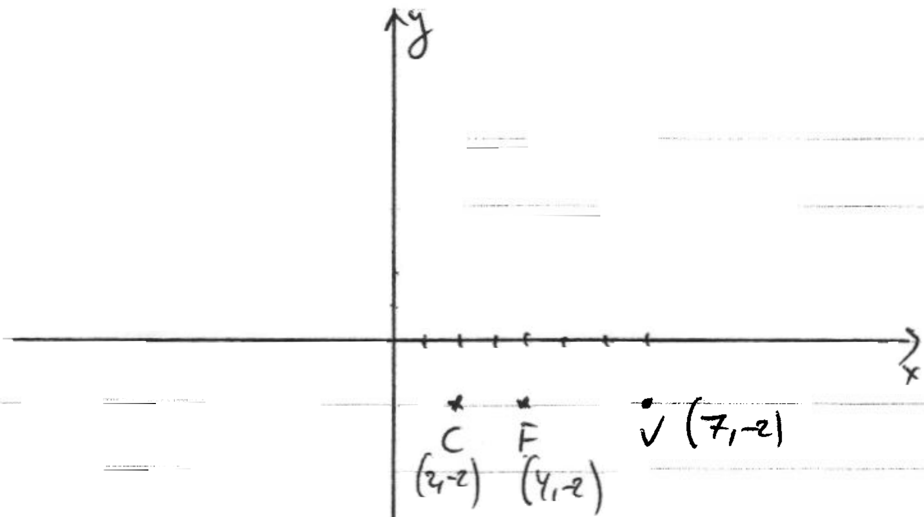
#55) Center $(2,-2)$, Vertex $(7,-2)$, Foci at $(4,-2)$



Here we have an ellipse whose major axis is // to the x-axis.

$$\frac{(x-2)^2}{a^2}+\frac{(y+2)^2}{b^2}=1$$

$a=$ Distance between Vertex and Center $=7-2=5$, $a^2=25$

$c=$ Distance between Focus and Center $=4-2=2$, $c^2=4$

$c^2=a^2-b^2 \longrightarrow b^2=a^2-c^2=25-4=21$

$\therefore$ Eq. of the Ellipse is:

$$\frac{(x-2)^2}{25}+\frac{(y+2)^2}{21}=1$$

Ellipse: Center $(2,-2)$

let $X=x-2$, $Y=y+2$

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From $(2,-2)$ we have:

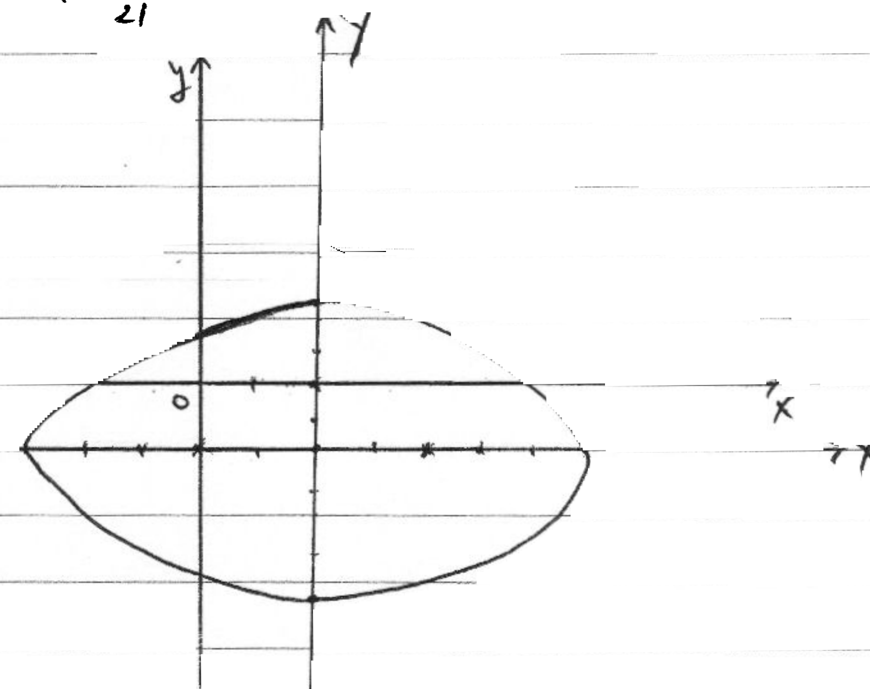
$$\frac{X^2}{25}+\frac{Y^2}{21}=1$$

Ellipse:

Center $(0,0)$

$a^2=25 \Rightarrow a=\pm 5$,

Major $(-5,0)$, $(5,0)$



$b^2=21 \Rightarrow b=\pm\sqrt{21}$, Minor $(0,-\sqrt{21})$, $(0,\sqrt{21})$

$c^2=a^2-b^2=25-21=4 \Rightarrow c=\pm 2$, Foci $(-2,0)$, $(2,0)$

Now from the original xy coordinate system, we have:

Ellipse: Center $(2,-2)$

Major $(-3,-2)$, $(7,-2)$

Minor $(2,-2-\sqrt{21})$, $(2,-2+\sqrt{21})$

Foci $(0,-2)$, $(4,-2)$.

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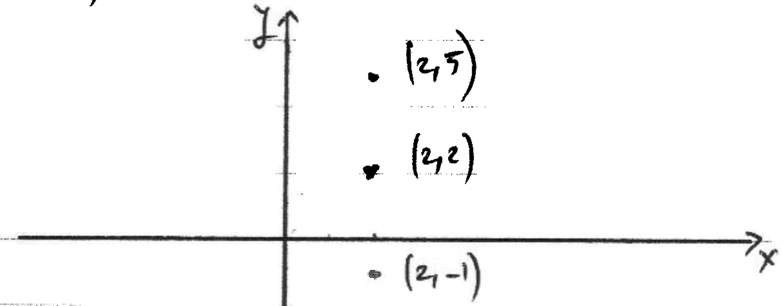
#60) Vertices at $(2,5)$ and $(2,-1)$, $c=2$



Center = Midpoint between vertices. $= (2,2)$

Eq. of Ellipse:

$$\frac{(x-2)^2}{b^2} + \frac{(y-2)^2}{a^2} = 1$$

$a =$ Distance From Vertex to Center $= 5-2 = 3$, $a^2=9$

$$c^2 = a^2 - b^2 \Rightarrow \underline{b^2 = a^2 - c^2} = 9-4 = 5$$

∴ Eq of Ellipse is: $\dfrac{(x-2)^2}{5} + \dfrac{(y-2)^2}{9} = 1$

let $X = x-2$, $Y = y-2$

From $(2,2)$ we have:

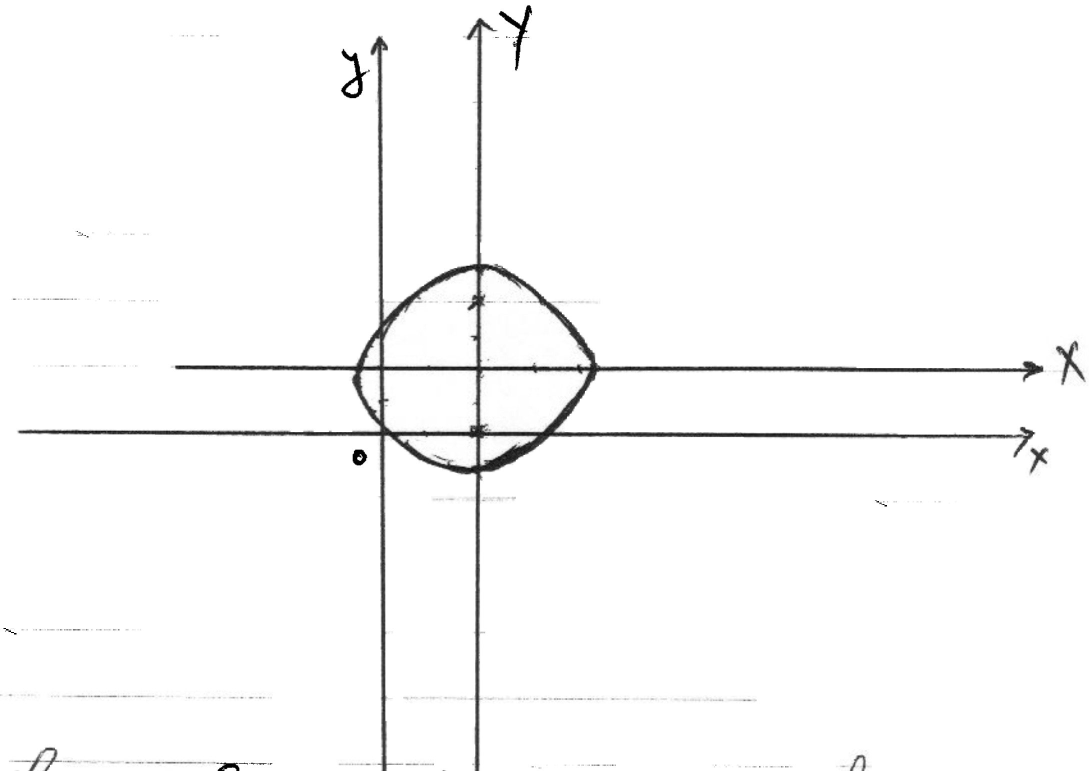
$$\frac{X^2}{5} + \frac{Y^2}{9} = 1$$

Ellipse: Center $(0,0)$

$a^2=9 \Rightarrow a = \pm 3$, Major $(0,-3)$, $(0,3)$

$b^2=5 \Rightarrow b = \pm\sqrt{5}$, Minor $(-\sqrt{5},0)$, $(\sqrt{5},0)$

$c^2 = a^2-b^2 = 9-5=4 \Rightarrow c = \pm 2$, Foci $(0,-2)$, $(0,2)$



From the original x-y coordinate system, we have:

An Ellipse:

Center $(2,2)$

Major $(2,-1)$, $(2,5)$

Minor $(2-\sqrt{5},-1)$, $(2+\sqrt{5},-1)$

Foci $(2,0)$, $(2,4)$