STUDY GUIDE FOR FINAL EXAMINATION
MATH 1113

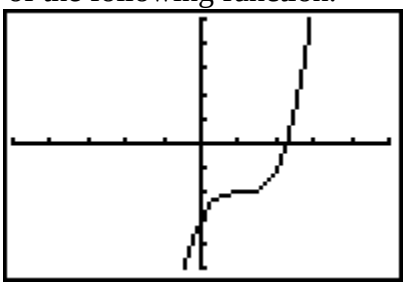
1. Let $f(x) = x^2 - 2x$. Compute the following.
   (a) $f(3)$ (b) $f(0)$
   (c) $f(a)$ (d) $f(a+2)$
   (e) $f(x+h)$ (f) $\dfrac{f(x+h)-f(x)}{h}$, $h \neq 0$

2. Find the domain of the following functions.
   (a) $f(x) = x^2 - 2x$ (b) $f(x) = \dfrac{1}{x+3}$
   (c) $f(x) = \sqrt{x+3}$ (d) $f(x) = \dfrac{1}{\sqrt{x+3}}$
   (e) $f(x) = e^x$ (f) $f(x) = \ln x$
   (g) $f(x) = \sin x$ (h) $f(x) = \tan x$
   (i) $f(x) = \csc x$ (j) $f(x) = \sin^{-1} x$
   (k) $f(x) = \tan^{-1} x$

3. Find a simplify $(f \circ g)(x)$.
   (a) $f(x) = \dfrac{3}{x+2}$; $g(x) = \dfrac{2}{3x}$ (b) $f(x) = 4x^2 + 2x + 8$; $g(x) = 2x - 3$

4. Find the inverse of the following functions.
   (a) $f(x) = \dfrac{1}{\sqrt[3]{3+x}}$ (b) $f(x) = x^3 + 3$

5. Sketch the graph of the inverse of the following function.



6. Put the following equations of conic sections in standard graphing form by completing the square. Then graph the conic sections. Give the following information for each conic section.
   Parabola: vertex, axis of symmetry, focus, and directrix.
   Ellipse: center, vertices, covertices, major axis, minor axis, foci, and eccentricity.
   Hyperbola: center, vertices, transverse axis, conjugate axis, foci, and eccentricity.
   (a) $4x^2 + y^2 - 16x + 2y + 13 = 0$
   (b) $x = 2y^2 + 2y + 3$
   (c) $9x^2 - 4y^2 + 36x + 8y - 4 = 0$

7. Find the equation, in standard form, of the parabola with vertex (7, −9) and focus (5, −9).

8. Find all the asymptotes of the graph of $R(x) = \dfrac{5x-2}{4x^2-7x}$.

9. Consider the function $R(x) = \dfrac{3x^2-5x-2}{x^2-4}$. The graph of R will have a vertical asymptote at _____ and will have a missing point at x = _____.

10. Find the oblique asymptote of the graph of $R(x) = \dfrac{x^2-17x+72}{x+3}$.

11. Suppose a radioactive substance Barnesvillium has a half-life of 77 years. If 258 grams are present initially, how much will be left after 212 years?

12. What is the half-life of a radioactive element that decays to 33% of the original amount present in 77 years?

13. Find the balance when$19,000 is invested at an annual rate of 6% for 5 years if the interest is compounded
   (a) daily
   (b) continuously.

14. How long will it take money to double at 17% interest compounded monthly?

15. (a) The angle of depression from a tower to a landmark is 19° and the landmark is 400 feet below the tower. How far away is the landmark?
   (b) Suppose the angle of elevation of the sun is 23.4°. Find the length of the shadow cast by Cindy Newman who is 5.75 feet tall.

16. If $\cos\theta = -\frac{4}{5}$ and $\frac{\pi}{2} \le \theta \le \pi$, find sec θ and cot θ.

17. Find where the graphs of the following functions have vertical asymptotes or horizontal asymptotes or state there are no asymptotes.
    (a) $y = \sin x$ (b) $y = \tan x$
    (c) $y = \csc x$ (d) $y = \cos^{-1} x$
    (e) $y = \tan^{-1} x$

18. Find the ***exact*** value of the following.
    (a) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (b) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
    (c) $\tan^{-1} 1$ (d) $\sin^{-1}\left(\sin\frac{5\pi}{4}\right)$
    (e) $\tan\left(\tan^{-1}\frac{1}{\pi}\right)$ (f) $\cos^{-1}\left[\cos\left(-\frac{\pi}{3}\right)\right]$

19. Solve the following trigonometric equations.
    (a) $2\sin x + 1 = 0$ (b) $2\cos^2 x + \sin x - 1 = 0$
    (c) $\cos 3x = -\frac{1}{2}$ (d) $\tan^3 x - \tan x = 0$

20. Approximate the solutions for each equation on the interval $0 \le x \le 2\pi$. Round your answer to four decimal places.
    (a) $\sin x = -0.739$ (b) $\cos x = 0.4201$
    (c) $\tan x = -5.17$

21. Establish the following trigonometric identities.
    (a) $\frac{\tan x}{\sin x} = \sec x$ (b) $\frac{\sin^2 t + \cos^2 t}{\tan t} = \cot t$
    (c) $\frac{\sin x - \cos x}{\sin x} = 1 - \cot x$ (d) $\frac{1-\cos\theta}{\tan^2\theta} = \frac{\cos^2\theta}{\cos\theta + 1}$
    (e) $\frac{\cos x - 1}{\sin x} = \cot x - \csc x$

22. Solve the following oblique triangles.
    (a) $A = 30°,\ b = 12,\ c = 16$ (b) $a = 3.55,\ b = 4.08,\ c = 2.83$
    (c) $B = 62.3°,\ C = 51.9°,\ c = 11.3$ (d) $C = 57.47°,\ b = 8.841,\ c = 0.777$
    (e) $A = 22°,\ a = 10,\ c = 15$

23. Find the area of the triangle ABC given $a = 8.5$, $B = 102°$, and $C = 27°$.

24. Convert the following polar equations into rectangular equations.
    (a) $r = \sqrt{7}$ (b) $r + 4\sin\theta = 0$
    (c) $r = 2\tan\theta$

25. Convert the following rectangular equations into polar equations.
    (a) $x + y = 1$ (b) $y^2 = 2x - x^2$
    (c) The circle centered at $(0, -4)$ with radius 4.

26. (a) Convert $\left(8, -\frac{3\pi}{4}\right)$ into rectangular coordinates.
    (b) Convert $\left(5\sqrt{3}, -5\right)$ into polar coordinates.

ANSWERS

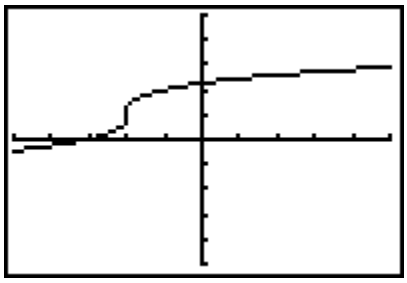
1. (a) 3 (b) 0
   (c) $a^2 - 2a$ (d) $a^2 + 2a$
   (e) $x^2 + 2xh + h^2 - 2x - 2h$ (f) $2x + h - 2$

2. (a) $(-\infty, \infty)$ (b) $(-\infty, -3) \cup (-3, \infty)$
   (c) $[-3, \infty)$ (d) $(-3, \infty)$
   (e) $(-\infty, \infty)$ (f) $(0, \infty)$
   (g) $(-\infty, \infty)$
   (h) all real numbers except $\frac{(2n+1)\pi}{2}$ where n is an integer; that is, all real numbers except the odd multiples of $\frac{\pi}{2}$
   (i) all real numbers except $2n\pi$ where n is an integer; that is, all real numbers except the even multiples of π
   (j) $[-1, 1]$ (k) $(-\infty, \infty)$

3. (a) $\frac{9x}{2 + 6x}$ (b) $16x^2 - 44x + 38$

4. (a) $f^{-1}(x) = \frac{1}{x^3} - 3$ (b) $f^{-1}(x) = \sqrt[3]{x - 3}$

5.



6. (a) ellipse; $(x-2)^2+\frac{(y+1)^2}{4}=1$; center: (2, −1); vertices: (2, 1), (2, −3); covertices: (1, −1), (3, −1); foci: $\left(2,-1+\sqrt{3}\right),\left(2,-1-\sqrt{3}\right)$; maj. axis: $x = 2$; min. axis: $y = -1$; $e=\frac{\sqrt{3}}{2}$

(b) parabola; $\left(y+\frac{1}{2}\right)^2=\frac{1}{2}\left(x-\frac{5}{2}\right)$; vertex: $\left(\frac{5}{2},-\frac{1}{2}\right)$; axis of symm: $y=-\frac{1}{2}$; focus: $\left(\frac{21}{8},-\frac{1}{2}\right)$; directrix: $x=\frac{19}{8}$

(c) hyperbola; $\frac{(x+2)^2}{4}-\frac{(y-1)^2}{9}=1$; center: (−2, 1); vertices: (0, 1), (−4, 1); foci: $\left(-2+\sqrt{13},1\right),\left(-2-\sqrt{13},1\right)$; trans. axis: $y = 1$; conj. axis: $x = -2$; $e=\frac{\sqrt{13}}{2}$

7. $(y+9)^2=-8(x-7)$

8. vertical asymptotes: $x = 0$, $x=\frac{7}{4}$; horizontal asymptote: $y = 0$

9. vertical asymptote at $x = -2$, missing point at $x = 2$

10. $y = x - 20$

11. 38.3 grams

12. 48.14 years

13. (a) $25,646.69 (b) $25,647.32

14. 4.12 years

15. (a) 1161.68 feet (b) 13.29 feet

16. $\sec\theta=-\frac{5}{4};\ \cot\theta=-\frac{4}{3}$

17. (a) no asymptotes

(b) vertical asymptotes at $x=\frac{(2n+1)\pi}{2}$ where n is an integer

(c) vertical asymptotes at $x = 2n\pi$ where n is an integer

(d) no asymptotes

(e) horizontal asymptotes: $y = -\frac{\pi}{2}$, $y = \frac{\pi}{2}$

18. (a) $-\frac{\pi}{3}$

(b) $\frac{3\pi}{4}$

(c) $\frac{\pi}{4}$

(d) $-\frac{\pi}{4}$

(e) $\frac{1}{\pi}$

(f) $\frac{\pi}{3}$

19. (a) $\left\{\frac{7\pi}{6} + k2\pi, \frac{11\pi}{6} + k2\pi, \text{ where } k \text{ is an integer}\right\}$

(b) $\left\{\frac{7\pi}{6} + k2\pi, \frac{11\pi}{6} + k2\pi, \frac{\pi}{2} + k2\pi, \text{ where } k \text{ is an integer}\right\}$

(c) $\left\{\frac{2\pi}{9} + \frac{k2\pi}{3}, \frac{4\pi}{9} + \frac{k2\pi}{3}, \text{ where } k \text{ is an integer}\right\}$

(d) $\left\{k\pi, \frac{\pi}{4} + k2\pi, \frac{3\pi}{4} + k2\pi, \frac{5\pi}{4} + k2\pi, \frac{7\pi}{4} + k2\pi, \text{ where } k \text{ is an integer}\right\}$

20. (a) 3.9731, 5.4516

(b) 1.1372, 5.1459

(c) 1.7619, 4.9036

21. The answer is in working the problems.

22. (a) $B \approx 46.94°$, $C \approx 103.06°$, $a \approx 8.21$

(b) $A \approx 58.54°$, $B \approx 78.62°$, $C \approx 42.84°$

(c) $A \approx 65.8°$, $a \approx 13$, $b \approx 13$

(d) no triangle

(e) $B_1 \approx 12.19°$, $C_1 \approx 145.81°$, $b_1 \approx 5.636$; $B_2 \approx 123.81°$, $C_2 \approx 34.19°$, $b_2 \approx 22.180$

23. ≈ 20.6 square units

24. (a) $x^2 + y^2 = 7$

(b) $x^2 + (y+2)^2 = 4$

(c) $x^4 + x^2y^2 - 4y^2 = 0$

25. (a) $r = \frac{1}{\cos\theta + \sin\theta}$

(b) $r = 2\cos\theta$

(c) $r = -8\sin\theta$

26. (a) $\left(-4\sqrt{2}, -4\sqrt{2}\right)$

(b) $\left(10, -\frac{\pi}{6}\right), \left(-10, \frac{5\pi}{6}\right)$, etc