Dr. ZABDAWI **Math Modeling**

Properties Of Logarithms

1) $y = a^x \iff x = \log_a y \quad ; \ a > 0 \ and \ a \neq 1 \quad (Napier's\ Definition\ 1614)$

2) $\log_a 1 = 0$, ex: log 1=0 , ln 1 = 0

3) $\log_a a = 1$, ex: log 10=1, ln e = 1

$$\log x = \log_{10} x \quad , \quad \ln x = \log_e x$$

4) $\log_a (x * y) = \log_a x + \log_a y \neq \log_a (x + y)$

$\log(x * y) = \log x + \log y \neq \log(x + y)$

$\ln(x * y) = \ln(x) + \ln(y)$

5) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y \neq \log_a (x - y)$

$\log\left(\frac{x}{y}\right) = \log x - \log y \neq \log(x - y)$

$\ln(\frac{x}{y}) = \ln(x) - \ln(y) \neq \ln(x - y)$

6) $\log_a x^r = r \log_a x \ : \ \log x^r = r \log x \quad , \quad \ln x^r = r \ln x$

7) $$\log_a x = \frac{\log_b x}{\log_b a}$$

Examples : $\log_5 7 = \frac{\ln 7}{\ln 5} = \frac{\log 7}{\log 5}$, $\log_{2.3} 8.69 = \frac{\ln 8.69}{\ln 2.3} = \frac{\log 8.69}{\log 2.3}$

$$\log x = \frac{\ln x}{\ln 10} \quad , \quad \ln x = \frac{\log x}{\log e}$$

Exponentials have a **very fast growth**, while logarithms have a **very slow growth.**