STRATEGY FOR GRAPHING POLYNOMIALS & RATIONAL FUNCTIONS

Dr. Marwan Zabdawi

Gordon College

[mzabdawi@gdn.edu](mailto:mzabdawi@gdn.edu)

81 Chad Court

McDonough, GA 30253

Abstract

Almost all books in College Algebra, Pre-Calc. and Calculus, do not give the student a specific outline on how to graph polynomials and rational functions. Instead, domains, intercepts, limits, continuity and asymptotes are detailed separately, and the student is left bewildered in a mathematical maze trying to find a way out. This paper uses all of the individual graphing ingredients and weaves them in a step by step procedure, where the student can go through it mechanically and without a hitch.

An interactive (bullet format) outline follows with two examples to demonstrate the procedure.

**Procedure:**

1. State the domain.
2. Find the Y-intercepts (x=0), and the X-Intercepts (y=0) the easy one in particular. **You can use the synthetic division to find the rational zeros for the given polynomial function. Basically, if f(c)=0, then (x-c) is a factor of f(x).**
3. For rational functions **ONLY**, find the asymptotes.
4. Perform the sign analysis.
5. Graph the function.

Now if we elaborate on step (3) for rational functions, we have: vertical asymptotes, horizontal asymptotes, and oblique/slant asymptotes.

**Asymptotes For Rational Functions**

1. **Vertical Asymptotes:**

Whatever makes the denominator zero is your vertical asymptote, as long as you do not have 0/0. Remember that 0/0 means that you have a hole in the graph.

1. **Horizontal & Slant asymptotes:**

Are the limits of the rational function as

**Horizontal & Slant asymptotes**

Consider the following rational function:

1. If the power of the numerator is the same as the power of the denominator (n=m), then the horizontal asymptote is y = the ratio of the leading coefficients of x,
2. If the power of the numerator is less than the power of the denominator (n<m), then the horizontal asymptote is y=0.
3. If the power of the numerator is greater than the power of the denominator by one degree (n=m+1), then the slant asymptote is y= the quotient of the division.

**Here the synthetic division can prove helpful when warranted.**

**Notice that for rational functions, it should be very obvious that you cannot have horizontal and slant asymptotes at the same time.**

**Using the Outlined Procedure Graph:**

1. Domain:
2. Y-Intercept: x=0
3. X-Intercepts: y=0
4. Sign Analysis:

- -2 1 3 +

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  |  | + |  | + |

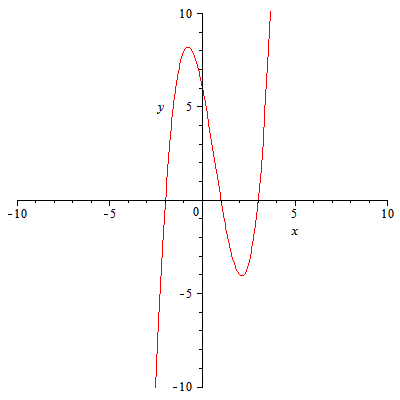


Fig. (1)

**Using the Outlined Procedure Graph:**

1. Domain:
2. Y-Intercept: x=0
3. X-Intercepts: y=0
4. Asymptotes:
5. Sign Analysis:

1 2

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  |  |  |  |  |  |

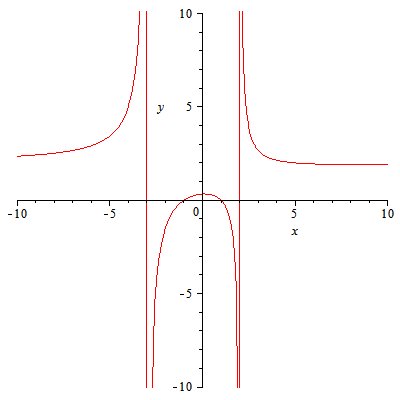


Fig.(2)