STRATEGY FOR GRAPHING POLYNOMIALS & RATIONAL FUNCTIONS

Dr. Marwan Zabdawi

Gordon College

mzabdawi@gdn.edu

81 Chad Court

McDonough, GA 30253

Abstract

Almost all books in College Algebra, Pre-Calc. and Calculus, do not give the student a specific outline on how to graph polynomials and rational functions. Instead, domains, intercepts, limits, continuity and asymptotes are detailed separately, and the student is left bewildered in a mathematical maze trying to find a way out. This paper uses all of the individual graphing ingredients and weaves them in a step by step procedure, where the student can go through it mechanically and without a hitch.

An interactive (bullet format) outline follows with two examples to demonstrate the procedure.

**Procedure:**

1. State the domain.
2. Find the Y-intercepts (x=0), and the X-Intercepts (y=0) the easy one in particular. **You can use the synthetic division to find the rational zeros for the given polynomial function. Basically, if f(c)=0, then (x-c) is a factor of f(x).**
3. For rational functions **ONLY**, find the asymptotes.
4. Perform the sign analysis.
5. Graph the function.

Now if we elaborate on step (3) for rational functions, we have: vertical asymptotes, horizontal asymptotes, and oblique/slant asymptotes.

**Asymptotes For Rational Functions**

1. **Vertical Asymptotes:**

Whatever makes the denominator zero is your vertical asymptote, as long as you do not have 0/0. Remember that 0/0 means that you have a hole in the graph.

1. **Horizontal & Slant asymptotes:**

Are the limits of the rational function as $x\rightarrow \pm \infty $

**Horizontal & Slant asymptotes**

Consider the following rational function:

$$f\left(x\right)=\frac{a\_{n}x^{n}+a\_{n-1}x^{n-1}+…+a\_{1}x+a\_{0}}{b\_{m}x^{m}+b\_{m-1}x^{m-1}+…+b\_{1}x+b\_{0}}$$

1. If the power of the numerator is the same as the power of the denominator (n=m), then the horizontal asymptote is y = the ratio of the leading coefficients of x, $y=\frac{a\_{n}}{b\_{m}}$
2. If the power of the numerator is less than the power of the denominator (n<m), then the horizontal asymptote is y=0.
3. If the power of the numerator is greater than the power of the denominator by one degree (n=m+1), then the slant asymptote is y= the quotient of the division.

**Here the synthetic division can prove helpful when warranted.**

**Notice that for rational functions, it should be very obvious that you cannot have horizontal and slant asymptotes at the same time.**

**Using the Outlined Procedure Graph:**

$$f\left(x\right)=(x-1)(x+2)(x-3)$$

1. Domain: $x\in \left(-\infty ,\infty \right)$
2. Y-Intercept: x=0 $\rightarrow \left(0,6\right)$
3. X-Intercepts: y=0$\rightarrow \left(1,0\right),\left(-2,0\right),\left(3,0\right)$
4. Sign Analysis:

 -$\infty $ -2 1 3 +$\infty $

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| $$x$$ |  |  |  |  |
| $$f(x)$$ | $$-$$ | + | $$-$$ | + |



 Fig. (1)

**Using the Outlined Procedure Graph:**

$$f\left(x\right)=\frac{2(x^{2}-1)}{\left(x+3\right)\left(x-2\right)}$$

1. Domain: $xϵ\left(-\infty ,-3\right)∪\left(-3,2\right)∪\left(2,\infty \right)$
2. Y-Intercept: x=0$\rightarrow \left(0,\frac{1}{3}\right)$
3. X-Intercepts: y=0$\rightarrow \left(-1,0\right), \left(1,0\right)$
4. Asymptotes:

$$x\rightarrow \pm \infty , y\rightarrow 2; y=2 is a Horizonatl Asymptote$$

$$x\rightarrow -3, y\rightarrow \pm \infty ; x=-3 is a Vertical Asymptote$$

$$x\rightarrow 2, y\rightarrow \pm \infty ; x=2 is a Vertical Asymptote$$

1. Sign Analysis:

 $ -\infty $ $-3$ $-1$ 1 2 $ \infty $

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| $$x$$ |  |  |  |  |  |
| $$f(x)$$ | $$+$$ | $$-$$ | $$+$$ | $$-$$ | $$+$$ |



Fig.(2)