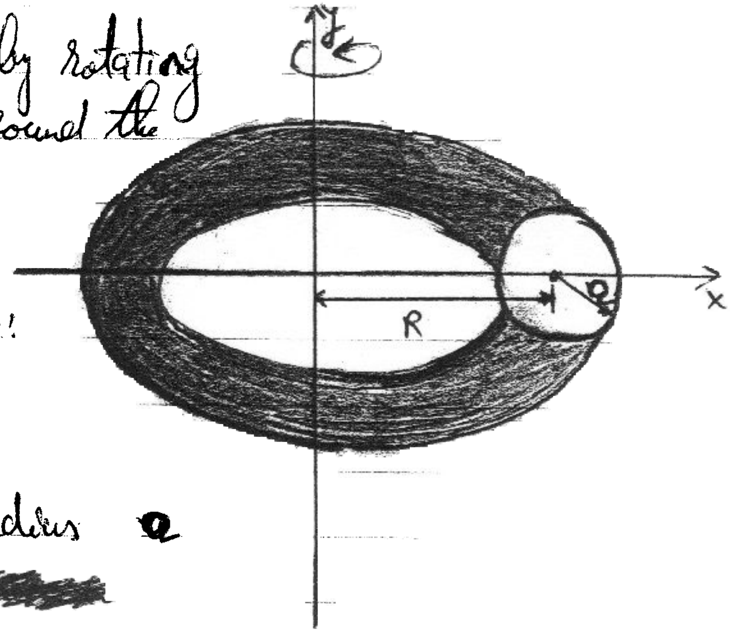
Dr. ZABIDAWI

The Volume of a Doughnut

The doughnut is made by rotating the circular cross section around the y-axis.



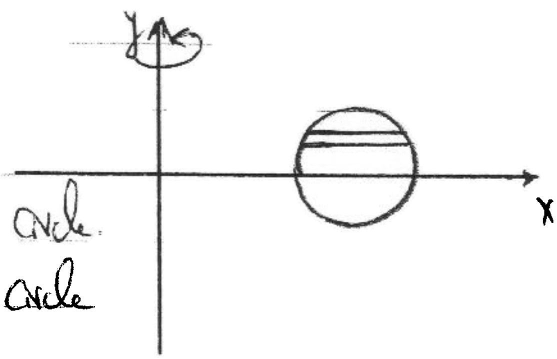
Eq. of the circular cross section:

$$(x-R)^2+y^2=a^2$$

Circle with center $(R,0)$ radius a

Washer Method:

$$\text{Volume} = \pi\int_{-a}^{a}(R^2-r^2)\,dy$$



R = Outer Radius = outer x value of circle

r = Inner Radius = Inner x value of circle

$$(x-R)^2+y^2=a^2$$

$$(x-R)^2=a^2-y^2$$

$$x-R=\pm\sqrt{a^2-y^2}$$

$$x=R\pm\sqrt{a^2-y^2}$$

Outer x-value $= R+\sqrt{a^2-y^2}$

Inner x-value $= R-\sqrt{a^2-y^2}$

$$\text{Volume} = 2\pi\int_0^a\left(\left[R+\sqrt{a^2-y^2}\right]^2-\left[R-\sqrt{a^2-y^2}\right]^2\right)dy$$

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$$\text{Volume} = 2\pi\int_0^a \left[R^2 + 2R\sqrt{a^2-y^2} + (a^2-y^2) - \left(R^2 - 2R\sqrt{a^2-y^2} + (a^2-y^2)\right)\right]dy$$

$$= 2\pi \int_0^a 4R\sqrt{a^2-y^2}\, dy$$

$$= 8\pi R \int_0^a \sqrt{a^2-y^2}\, dy$$

But $\int_0^a \sqrt{a^2-y^2}\, dy = \frac{1}{4}$ Area of a circle of Radius $= a$

$$\text{Volume} = 8\pi R \cdot \frac{\pi a^2}{4} = 2\pi^2 R a^2$$

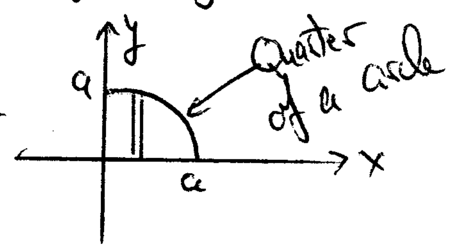
where R = Radius of the donut

a = Radius of cross section of doughnut.

Because if we have:

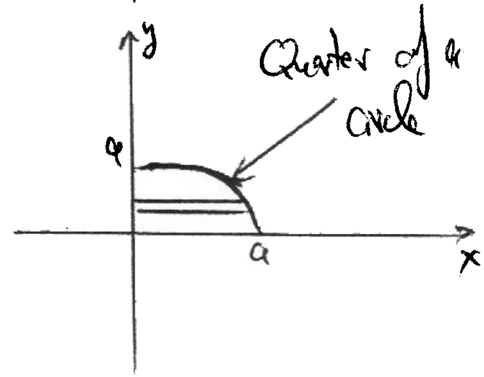
$x^2+y^2 = a^2$

$x = \pm\sqrt{a^2-y^2}$ or $y = \pm\sqrt{a^2-x^2}$



$$\text{Area} = \int_0^a y\, dx = \int_0^a \sqrt{a^2-x^2}\, dx = \frac{1}{4}\pi a^2$$

OR



$$\text{Area} = \int_0^a x\, dy = \int_0^a \sqrt{a^2-y^2}\, dy$$

$= \frac{1}{4}$ Area of a circle

$= \frac{1}{4}\pi a^2$