Dr. ZABDAWI

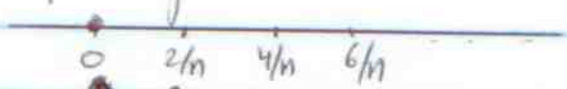
SPECIAL PROBLEM

Find the area under the parabola $y = x^2 + 1$ over $[0,2]$

$$\text{Area} = \lim_{n \to \infty} \sum f(x_i^*)\,\Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n}$$

a) Height is based on the right endpoint of the subinterval.



$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\,\frac{2}{n} \quad ; \quad f(x) = x^2 + 1$$

R.E.P $\Rightarrow$ $x_i^* = a + \Delta x\, i = 0 + \frac{2}{n} i = \frac{2}{n} i$

$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} \left[\left(\frac{2}{n} i\right)^2 + 1\right] \frac{2}{n}$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{4}{n^2} i^2 + 1\right)\frac{2}{n} = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{8}{n^3} i^2 + \frac{2}{n}\right)$$

$$= \lim_{n \to \infty} \frac{8}{n^3} \sum_{i=1}^{n} i^2 + \lim_{n \to \infty} \sum_{i=1}^{n} \frac{2}{n}$$

$$= \lim_{n \to \infty} \left(\frac{8}{n^3} \frac{n(n+1)(2n+1)}{6} + \frac{2}{n} n\right)$$

$$= \frac{8}{3} + 2 = \frac{14}{3} \text{ Square Units.}$$

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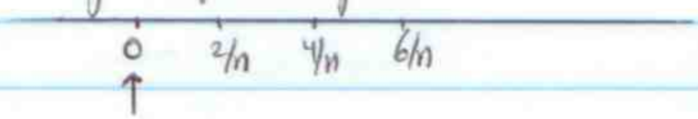
b) Height is based on the left endpoint of the subinterval



$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \frac{2}{n} \quad ; \quad f(x) = x^2+1$$

<u>**L.E.P.:**</u> $\quad x_i^* = a + \Delta x (i-1) = 0 + \frac{2}{n}(i-1)$

$$f(x_i^*) = \left[\frac{2}{n}(i-1)\right]^2 + 1$$

$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{n} \left(\left[\frac{2}{n}(i-1)\right]^2 + 1\right)\frac{2}{n}$$

$$= \lim_{n \to \infty} \sum_{i=1}^{n} \left(\frac{4}{n^2}(i-1)^2 + 1\right)\frac{2}{n}$$

$$= \lim_{n \to \infty} \frac{2}{n} \sum_{i=1}^{n} \left[\frac{4}{n^2}\left(i^2 - 2i + 1\right) + 1\right]$$

$$= \lim_{n \to \infty} \frac{2}{n} \left[\frac{4}{n^2}\left(\frac{n(n+1)(2n+1)}{6} - 2\,\frac{n(n+1)}{2} + n\right) + n\right]$$

$$= \frac{2 \times 4 \times 2}{6} - 0 + 0 + 2 = \frac{8}{3} + 2$$

$$= \frac{14}{3} \text{ Square Units}$$

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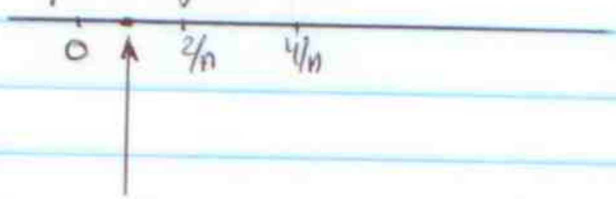
c) Height is based on the midpoint of the subinterval



$f(x) = x^2+1$

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x \quad ; \quad \Delta x = \frac{2}{n}$$

Midpoint: $x_i^* = a + \Delta x\left(\frac{2i-1}{2}\right) = 0 + \frac{2}{n}\frac{(2i-1)}{2} = \frac{(2i-1)}{n}$

$$\Rightarrow f(x_i^*) = \left(\frac{2i-1}{n}\right)^2 + 1$$

$$\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n}\left[\left(\frac{2i-1}{n}\right)^2+1\right]\frac{2}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n}\left[\frac{(4i^2-4i+1)}{n^2}+1\right]\frac{2}{n}$$

$$= \lim_{n\to\infty} \frac{2}{n}\sum_{i=1}^{n}\left[\frac{1}{n^2}\left(4i^2-4i+1\right)+1\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[\frac{1}{n^2}\left(4\frac{n(n+1)(2n+1)}{6} - 4\frac{n(n+1)}{2} + n\right) + n\right]$$

$$= \lim_{n\to\infty} \frac{2}{n}\left[\frac{1}{n^2}\left(\frac{2n(n+1)(2n+1)}{3} - 2n(n+1) + n\right) + n\right]$$

$$= \frac{8}{3} - 0 + 0 + 2 = \frac{14}{3} \text{ Square Units}$$