10/22/14

1) $y = \dfrac{x^2}{x^2-9}$

1. Domain $x^2-9 \neq 0$

$x^2 \neq 9 \Longrightarrow x = \pm 3$

D: $x \in (-\infty,-3) \cup (-3,3) \cup (3,\infty)$

2. $x=0 \Longrightarrow (0,0)$

$y=0 \Longrightarrow (0,0)$: $\dfrac{x^2}{x^2-9} = 0 \Longrightarrow \boxed{x=0}$

3. Asymptotes.

$$\left.\begin{array}{l} \text{As } x \longrightarrow -3 \text{ ; } y \longrightarrow \infty \\ \text{As } x \longrightarrow +3 \text{ , } y \longrightarrow \infty \end{array}\right\} \therefore x = \pm 3 \text{ are Vertical Asymptotes.}$$

It is relevant to do $x \longrightarrow -3^-$ or $x \longrightarrow -3^+$ because the sign analysis will show whether $\pm\infty$

$$\lim_{x\to\pm\infty} \frac{x^2}{x^2-9} = \lim_{x\to\pm\infty} \frac{x^2/x^2}{x^2/x^2 - 9/x^2} = 1$$

$\therefore$ $y=1$ is a horizontal asymptote.

4. $$\frac{dy}{dx} = \frac{2x(x^2-9) - 2x \cdot x^2}{(x^2-9)^2} = \frac{2x^3-18x-2x^3}{(x^2-9)^2}$$

$$\frac{dy}{dx} = y' = \frac{-18x}{(x^2-9)^2}$$

$$y' = \frac{-18x}{(x^2-9)^2}$$

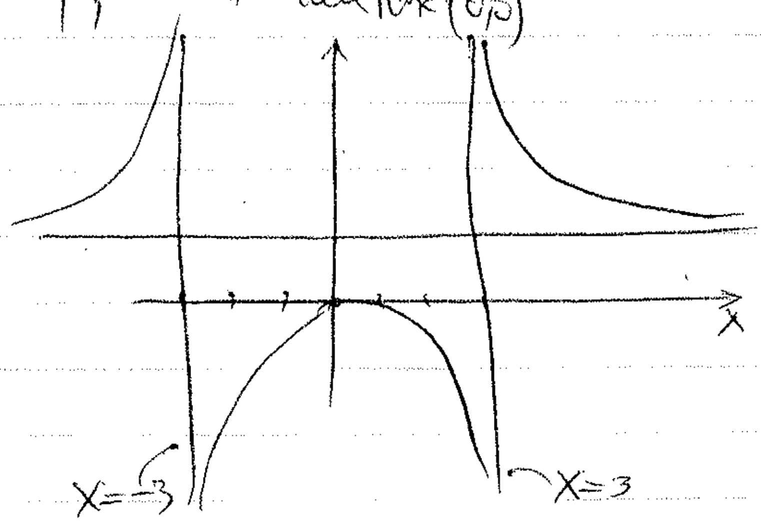
Critical Pts Numerator $= 0 \implies x = 0$

Denominator $= 0 \implies x = \pm 3$

Sign Analysis for y'

x	$-\infty$		-3		0		3		$+\infty$
y'		$+$		$+$	0	$-$		$-$	
y		$\nearrow$		$\nearrow$	local Max $(0,0)$	$\searrow$		$\searrow$	

By test Points.



$$y'' = \frac{-18(x^2-9)^2 - 2(x^2-9)(2x)(-18x)}{(x^2-9)^4}$$

$$= \frac{-18(x^2-9)\left[(x^2-9) - 4x^2\right]}{(x^2-9)^4} = \frac{-18(-3x^2-9)}{(x^2-9)^3}$$

$$= \frac{54(x^2+3)}{(x^2-9)^3}$$

$$y'' = \frac{54(x^2+3)}{(x^2-9)^3}$$

Sign Analysis for y'':

Numerator $\neq 0$

Denominator $(x^2-9)^3 = 0 \implies x = \pm 3$

Sign Analysis for y''

x	$-\infty$ $\quad -3$	$\quad 3$	$\quad\infty$
y''	+	−	+
y is Concaved	UP	DOWN	UP

2) $y = \dfrac{x-2}{x^4}$

1. Domain $x \neq 0$

   D: $x \in (-\infty, 0) \cup (0, \infty)$

2. Intercepts $x \neq 0$

   $y = 0 \implies x = 2,\ (2, 0)$

3. Asymptotes:

   As $x \to \pm\infty$, $y \to 0$; $y = 0$ Asymptote

   As $x \to 0$, $y \to \infty$ $x = 0$ A

4. $y' = \dfrac{dy}{dx} = \dfrac{1 \cdot x^4 - 4x^3(x-2)}{x^8} = \dfrac{x^4 - 4x^4 + 8x^3}{x^8}$

$$\frac{dy}{dx} = \frac{x^3(-3x+8)}{x^8}\ ; \quad x \neq 0$$

$$\frac{dy}{dx} = \frac{8-3x}{x^5}$$

$$\frac{dy}{dx} = \frac{8-3x}{x^5}$$

Critical Pts are Numerator $= 0 \Longrightarrow x = 8/3$

Denominator $= 0 \Longrightarrow x = 0$

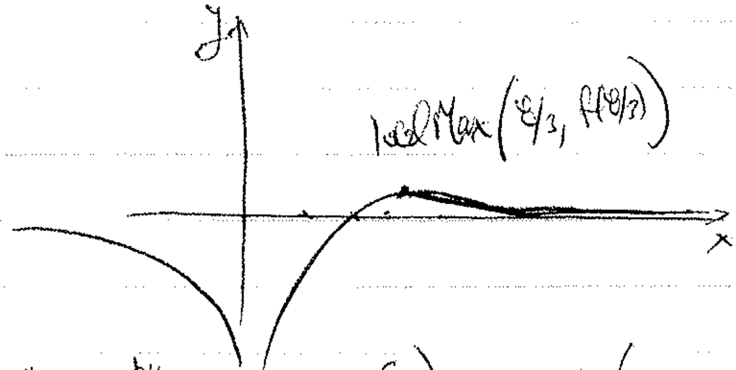
Sign Analysis of y'

x	$-\infty$	0		$8/3$	$+\infty$
y'	$-$	$\Vert$	$+$	0	$-$
y	$\searrow$	$\Vert$	$\nearrow$		$\searrow$

By test Pts

local Max $\left(8/3, f(8/3)\right)$

$(2.67, .0132)$



$$y'' = \frac{d^2y}{dx^2} = \frac{-3(x^5) - 5x^4(8-3x)}{(x^5)^2}$$

$$= \frac{-3x^5 - 40x^4 + 15x^5}{x^{10}}$$

$$= \frac{-40x^4 + 12x^5}{x^{10}}$$

$$= \frac{4x^4(-10+3x)}{x^{10}} \quad ; \; x \neq 0$$

$$y'' = \frac{4(-10+3x)}{x^6}$$

$$y'' = \frac{4(3x-10)}{x^6}$$

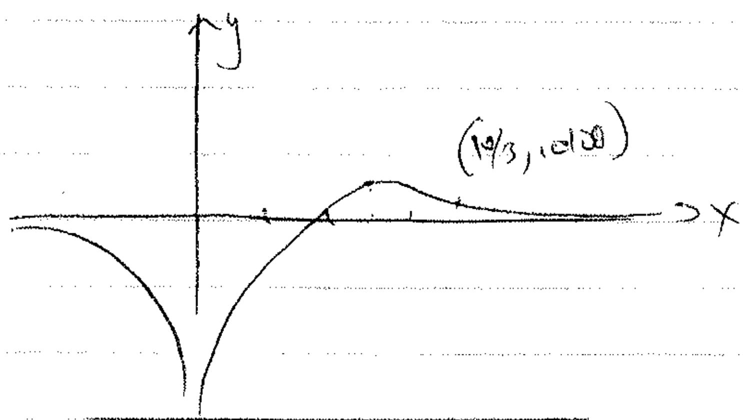
Critical Pts : Numerator $= 0 \Longrightarrow x = 10/3$

Denominator $= 0 \Longrightarrow x = 0$

Sign Analysis of y''

x	$-\infty$	0		$10/3$	$+\infty$
y''	$-$	‖	$-$	0	$+$
y is concave	$\downarrow$	‖	$\downarrow$		$\uparrow$

Inflection Pt $(10/3, f(10/3)) = (3.33, .1008)$



$$y = \frac{x^2+12}{x-2}$$

1. Domain $x - 2 \neq 0$; $x \neq 2$

D: $x \in (-\infty, 2) \cup (2, \infty)$

2. $x = 0$, $(0, -6)$

$y \neq 0$

3. $$y' = \frac{2x(x-2) - 1(x^2+12)}{(x-2)^2}$$

$$= \frac{2x^2 - 4x - x^2 - 12}{(x-2)^2}$$

$$= \frac{x^2 - 4x - 12}{(x-2)^2}$$

$$\frac{dy}{dx} = \frac{x^2-4x-12}{(x-2)^2} = \frac{(x-6)(x+2)}{(x-2)^2}$$

Critical Pts. Numerator $= 0 \longrightarrow x = 6, -2$

Denominator $= 0 \longrightarrow x = 2$

Sign Analysis on y'

x	$-\infty$		-2		2		6		∞
y'		$+$	0	$-$		$-$	0	$+$	
y		$\nearrow$		$\searrow$		$\searrow$		$\nearrow$	

local Max $(-2, -4)$ local Min $(6, 12)$

Asymptotes : As $x \longrightarrow 2$, $y \longrightarrow \infty$

As $x \longrightarrow \pm\infty$; we have a slant asymptote

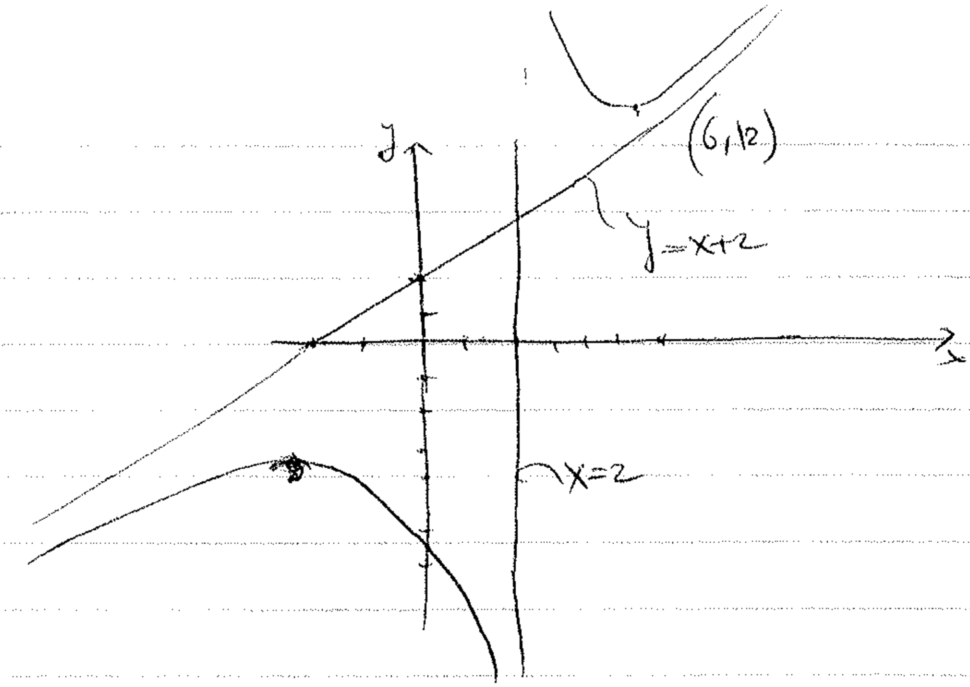
$$\begin{array}{r|l} & x + 12 \\ \hline x-2 & x^2 + 12 \\ & -x^2 + 12x \\ \hline & 12x + 12 \\ & -12x + 24 \\ \hline & 36 \end{array}$$

$$\therefore \frac{x^2+12}{x-2} = (x+12) + \frac{36}{x-2}$$

As $x \longrightarrow \pm\infty$ $y \longrightarrow x+12$

$\therefore$ $y = x+2$ is an oblique asymptote

Graph:



$$y' = \frac{x^2-4x-12}{(x-2)^2}$$

$$y'' = \frac{(2x-4)(x-2)^2 - 2(x-2)(1)(x^2-4x-12)}{(x-2)^4}$$

$$= \frac{(x-2)\left[(2x-4)(x-2) - 2(x^2-4x-12)\right]}{(x-2)^4}$$

$$= \frac{2x^2 - 8x + 8 - 2x^2 + 8x + 24}{(x-2)^3}$$

$$y'' = \frac{32}{(x-2)^3}$$

Critical Pts are Denominator $= 0$, $x = 2$

Sign Analysis of y''

x	$-\infty$	2	∞
y''	$-$		$+$
y is Concaved	$\downarrow$		$\uparrow$

No Inflection Pts.