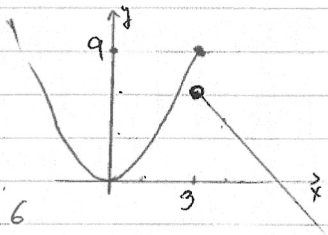
Study Guide For FE MATH 1501 — D. ZABDAWI

1) a) $\lim\limits_{x\to 1} \dfrac{x^2+7x-8}{x-1} = \lim\limits_{x\to 1} \dfrac{(x+8)(x-1)}{(x-1)} = \lim\limits_{x\to 1} (x+8) = 9$

b) $\lim\limits_{x\to\infty} \dfrac{\sin x}{x^2} = 0 \quad ; -1 \le \sin x \le 1$




c) $\lim\limits_{x\to 3} f(x)$, where $f(x) = \begin{cases} x^2 & \text{if } x \le 3 \\ 9-x & \text{if } x > 3 \end{cases}$

$\lim\limits_{x\to 3^-} f(x) = 9 \;,\; \lim\limits_{x\to 3^+} f(x) = 6$

$\Longrightarrow \lim\limits_{x\to 3} f(x) = DNE.$

d) $\lim\limits_{x\to -1} \dfrac{x^3}{(x+1)^2} = -\infty$

e) $\lim\limits_{x\to 0} \dfrac{\sin 3x}{\frac{x}{3}} = 3\lim\limits_{x\to 0} \dfrac{\sin 3x}{x} = 3\lim\limits_{x\to 0} 3\,\dfrac{\sin 3x}{3x}$

$= 9 \lim\limits_{x\to 0} \dfrac{\sin 3x}{3x} = 9$

$\lim\limits_{x\to 2^+} \dfrac{x+2}{|x+2|} = +1 \;;\; \lim\limits_{x\to 2^-} \dfrac{x-2}{|x-2|} = -1$

$\lim\limits_{x\to -2^-} \dfrac{x+2}{|x+2|} = -1$

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2) $f(x) = 2x^2 - 3x + 6$

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{2(x+h)^2 - 3(x+h) + 6 - (2x^2 - 3x + 6)}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 3x - 3h + 6 - 2x^2 + 3x - 6}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h + 6 - 2x^2 + 3x - 6}{h}$$

$$= \lim_{h \to 0} \frac{h(4x + 2h - 3)}{h} = \lim_{h \to 0} 4x + 2h - 3 = 4x - 3.$$

5) $f(x) = \sin(x^3)$

$$f'(x) = \frac{df}{dx} = \cos x^3 \cdot [3x^2]$$

$$= 3x^2 \cos x^3$$

6) $f(x) = \cos[\cos(\pi x)] = \cos u \quad ; \quad u = \cos \pi x$

$$\Rightarrow f' = \frac{df}{dx} = -\sin[\cos(\pi x)] \cdot -\sin(\pi x) \cdot \pi$$

$$= \pi \sin \pi x \cdot \sin[\cos(\pi x)]$$

$$\left.\frac{df}{dx}\right|_{x=1.1} = \pi \sin \pi x \cdot \sin[\cos(\pi x)]\Big|_{x=1.1} = 0.7903$$

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7) Find the equation of the tangent line to $x^2+9xy+y^2=36$ at the point $(0,6)$

Eq. of tangent line: $(y-6) = \left.\frac{dy}{dx}\right|_{(0,6)} (x-0)$

$$x^2+9xy+y^2=36$$

$$2x\,dx+9(y\,dx+x\,dy)+2y\,dy=0$$

$$2x+9\left(y+x\frac{dy}{dx}\right)+2y\frac{dy}{dx}=0$$

$$\frac{dy}{dx}(9x+2y) = -2x-9y$$

$$\frac{dy}{dx} = \frac{-2x-9y}{9x+2y} \Rightarrow \left.\frac{dy}{dx}\right|_{(0,6)} = \frac{-9\times 6}{2\times 6} = -\frac{9}{2}$$

$\rightarrow$ Eq. of tangent line is

$$y-6 = -\frac{9}{2}x$$

$$y = -\frac{9}{2}x+6.$$

$$\rightarrow \quad L(x) = -\frac{9}{2}x+6.$$

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8) Find the maximum value of $f(x) = x^3 + 2x^2 - 4x$ on $[-3, 1]$

$$\frac{df}{dx} = 3x^2 + 4x - 4$$
$$= (3x - 2)(x + 2)$$

Critical values are the x-values that make $\frac{df}{dx} = 0, \pm\infty$

$$\frac{df}{dx} = 0 \implies (3x-2)(x+2) = 0$$
$$\implies x = 2/3, \ -2$$

| $x$ | $-\infty$ | $-2$ | | $2/3$ | $\infty$ |
|---|---|---|---|---|---|
| $df/dx$ | $+$ | $0$ | $-$ | $0$ | $+$ |
| $f(x)$ | $\nearrow$ | | $\searrow$ | | $\nearrow$ |
| | | Max | | Min | |

$$f(-2) = (-2)^3 + 2(-2)^2 - 4(-2) = -8 + 8 + 8 = 8$$

$\therefore$ Max $= 8$ occurs at $(-2, 8)$.

9) Gravity on Mars $= 3.72\ m/sec^2$. If a rock is thrown straight up from the surface of Mars with an initial velocity of $23\ m/sec$. How high will it go ??

$g = -3.72 \downarrow$ ; $y$ ; $0$

acceleration $= -3.72$

$$\frac{dv}{dt} = -3.72$$

$$dv = -3.72\, dt$$

$$\int dv = \int -3.72\, dt$$

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#9)

$$\int dV = \int -3.72\, dt$$

$$V = -3.72\, t + C$$

@ $t=0$, $V(0) = 23$ m/sec

$$V(t) = -3.72t + 23.$$

At Maximum height $V(t)=0$

$$\Longrightarrow -3.72t + 23 = 0 \Longrightarrow t = \frac{-23}{-3.72} = 6.183 \text{ sec}.$$

$$\frac{ds}{dt} = V$$

$$ds = V\,dt$$

$$\int ds = \int (-3.72t + 23)\, dt$$

$$S = -3.72\frac{t^2}{2} + 23t + C \quad , @ t=0,\ S=0$$

$$\Longrightarrow C = 0$$

$$S(t) = -\frac{3.72}{2}t^2 + 23t$$

$$\text{Maximum Height} = S(t)\Big|_{t=6.183\text{ sec}} = -\frac{3.72}{2}(6.183)^2 + 23 \times 6.183$$

$$= 71.102 \text{ meters}.$$

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10) Find the maximum and minimum values of $f(x) = (x-3)^{2/3}$ on $[0,4]$

$$f(x) = (x-3)^{2/3}$$
$$\frac{df}{dx} = \frac{2}{3}(x-3)^{-1/3} = \frac{2}{3(x-3)^{1/3}}$$

Critical values are the x-values that make $\frac{df}{dx} = 0, \pm\infty$

$\Longrightarrow$ $x = 3$ is a critical value.

| $x$ | $-\infty$ | 3 | $+\infty$ |
|---|---|---|---|
| $df/dx$ | $-$ | | $+$ |
| $f(x)$ | $\searrow$ | | $\nearrow$ |

Minimum @ $(3,0)$

Minimum $= 0$ occurs at $(3,0)$

Check end points.

$x=0$, $f(0) = (-3)^{2/3} = \sqrt[3]{(-3)^2} = 9^{1/3}$

$x=4$, $f(4) = 1^{2/3} = 1$

$\longrightarrow$ Absolute Minimum $= 0$ occurs @ $(3,0)$

Absolute Maximum $= 9^{1/3}$ occurs @ $(0, 9^{1/3})$.

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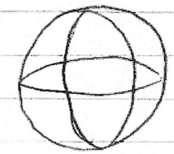
You do #11, #12.

#13) A spherical balloon is being blown up at a rate of 100 $cm^3/min$. At what rate is the radius r changing when r is 4 cm.



$$V = \frac{4}{3}\pi R^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3R^2 \frac{dR}{dt}$$

$$4\pi R^2 \frac{dR}{dt} = 100$$

$$\Longrightarrow \frac{dR}{dt} = \frac{100}{4\pi R^2}$$

$$\Longrightarrow \left.\frac{dR}{dt}\right|_{R=4} = \frac{100}{4\pi (4)^2} = 0.497 \text{ cm/min.}$$

#14) Given $x = \cos(xy)$ Find $y'\left(\frac{\sqrt{2}}{2}\right)$

$$x = \cos(xy)$$

$$dx = -\sin(xy) \cdot [dx \cdot y + x\,dy]$$

$$1 = -\sin(xy) \cdot \left[y + x\frac{dy}{dx}\right]$$

$$1 = -y\sin(xy) - x\sin(xy)\frac{dy}{dx}$$

$$(x\sin xy)\frac{dy}{dx} = -y\sin(xy) - 1$$

$$\rightarrow \frac{dy}{dx} = -\frac{(y\sin(xy)+1)}{x\sin(xy)}$$

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#14 Continue
$$\frac{dy}{dx} = -\frac{\left(y\sin(xy)+1\right)}{x\sin(xy)}$$

@ $x = \frac{\sqrt{2}}{2}$ we have $x = \cos(xy)$

$$\frac{\sqrt{2}}{2} = \cos(xy)$$
$$\Rightarrow xy = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$
$$\Rightarrow y = \frac{\pi}{4} \cdot \frac{2}{\sqrt{2}} = \frac{\pi}{2\sqrt{2}}$$

$$\frac{dy}{dx} = -\frac{\left(\frac{\pi}{2\sqrt{2}}\sin(\pi/4)+1\right)}{\frac{\sqrt{2}}{2}\cdot\sin(\pi/4)}$$

$$= -\frac{\left(\frac{\pi}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{2}+1\right)}{\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{2}}{2}}$$

$$= -\frac{\left(\pi/4+1\right)}{1/2}$$

$$= -2\left(\frac{\pi}{4}+1\right) = \boxed{-\frac{\pi}{2}-2}$$

$$= -3.571$$

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#15) MVT. find $c$ for $f(x) = \sin(3x)$ on $[0, \pi/4]$

M.V.T $f(x)$ is continuous on $[0, \pi/4]$
and differentiable on $(0, \pi/4)$

$$\Rightarrow \exists, c \in (0, \pi/4) \;/\; f'(c) = \frac{f(\pi/4) - f(0)}{\pi/4 - 0}$$

$$\frac{df}{dx} = 3\cos(3x)$$

$$\sin(3\pi/4) = \sin(\pi - \pi/4) = \sin \pi/4 = \frac{\sqrt{2}}{2}$$

$$\sin 0 = 0$$

$$\Rightarrow 3\cos(3c) = \frac{(\sqrt{2}/2 - 0)}{\pi/4 - 0}$$

$$= \frac{2\sqrt{2}}{\pi}$$

$$\cos(3c) = \frac{2\sqrt{2}}{3\pi}$$

$$3c = \cos^{-1}\left(\frac{2\sqrt{2}}{3\pi}\right) = 1.266$$

$$c = \frac{1.266}{3} = 0.422$$

$$\boxed{c = 0.422}$$

You do #16

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Compute the following:

17) a) $\int \frac{x^2+1}{x^2}\,dx = \int \left(1+\frac{1}{x^2}\right)dx = x + \frac{x^{-1}}{-1} + c = 1 - \frac{1}{x} + c$

b) $\int x\sqrt{x^2+4}\,dx$

let $u = x^2+4 \implies du = 2x\,dx$

$x\,dx = \frac{1}{2}\,du$

$\frac{1}{2}\int u^{1/2}\,du = \frac{1}{2}u^{3/2}\cdot\frac{2}{3} + c = \frac{1}{3}\left(x^2+4\right)^{3/2} + C$

c) $\int x^2\cos^5\left(x^3+5\right)\sin\left(x^3+5\right)dx$

let $u = \cos\left(x^3+5\right) \implies du = -\sin\left(x^3+5\right)\cdot 3x^2\,dx$

$\implies x^2dx\,\sin\left(x^3+5\right) = -\frac{1}{3}\,du$

$I = -\frac{1}{3}\int u^5\,du = -\frac{1}{3}\frac{u^6}{6} + C = -\frac{1}{18}\cos^6\left(x^3+5\right) + C$

18) a) $\int_{-\pi/2}^{\pi/2}\left(2x+\cos x\right)dx = \left(x^2+\sin x\right)\Big|_{-\pi/2}^{\pi/2}$

$= \left(\frac{\pi^2}{4}+1\right) - \left(\frac{\pi^2}{4}-1\right) = 2$

b) $\int_0^2 \frac{t^3}{\sqrt{t^4+9}}\,dt$ Let $u = t^4+9$

$\implies du = 4t^3\,dt \implies t^3dt = \frac{1}{4}\,du$

$\frac{1}{4}\int_9^{25}\frac{du}{u^{1/2}} = \left(\frac{1}{4}u^{1/2}\cdot\frac{2}{1}\right)\Big|_9^{25} = \frac{1}{2}\left(5-3\right) = 1$

You can do (c) and (d)

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#19) a) $g(x) = \int^{x} \sqrt[4]{1+t^4}\, dt \Rightarrow \frac{dg}{dx} = \sqrt[4]{1+x^4}$ FTC I

b) $g(x) = \int_1^{\sqrt{x}} \frac{t^2}{(t^2+1)}\, dt$

$\frac{dg}{dx} = \frac{x}{(x+1)} \cdot \frac{1}{2} x^{-1/2} = \frac{x}{2\sqrt{x}\cdot(x+1)}$.

c) $g(x) = \int_{x^2-3x} \left(t + \sin t\right) dt \Rightarrow -\left(x^2-3x+\sin(x^2-3x)\right)(2x-3) = \frac{dg}{dx}$

$= (3-2x)\left(x^2-3x+\sin(x^2-3x)\right)$

d) $g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}}\, dt \Rightarrow \frac{1}{\sqrt{2+x^8}} \cdot 2x - \frac{1}{\sqrt{2+\tan^4 x}} \cdot \sec^2 x = \frac{dg}{dx} =$

$= \frac{2x}{\sqrt{2+x^8}} - \frac{\sec^2 x}{\sqrt{2+\tan^4 x}}$

You do 20 and 21 "They were H.W. problems"

#22) Find $\frac{dy}{dx}$ for $y = 5\sin^4(x^3-3x^2)$

Use the chain rule $y = 5\left[\sin(x^3-3x^2)\right]^4 = 5U^4$

$\frac{dy}{dx} = \frac{dy}{dU} \cdot \frac{dU}{dx} = 20\sin^3(x^3-3x^2) \cdot \cos(x^3-3x^2)(3x^2-6x)$

$= 60(x^2-2x)\sin^3(x^3-3x^2)\cos(x^3-3x^2)$

$= 60x(x-2)\sin^3(x^3-3x^2)\cos(x^3-3x^2)$.

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#23) Verify the following inequalities without evaluating the integrals.

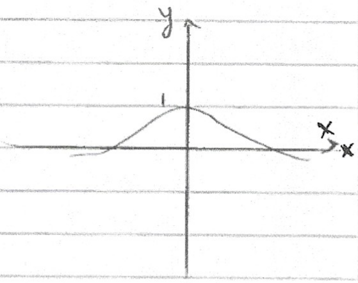
a) $\int_0^1 x^2 \cos x \, dx \leq 1/3$

$$-1 \leq \cos x \leq 1$$

$$-x^2 \leq x^2 \cos x \leq x^2$$

$$\int_0^1 x^2 \cos x \, dx \leq \int_0^1 x^2 dx = \frac{x^3}{3}\Big|_0^1 = 1/3 \quad V_4.$$

b) $\int_{\pi/4}^{\pi/2} \frac{\sin x}{x} dx \leq \frac{\sqrt{2}}{2}$




$$\frac{\sin x}{x}\Big|_{x=\pi/2} = 1 \cdot \frac{2}{\pi}$$

$$\frac{\sin x}{x}\Big|_{x=\pi/4} = \frac{\sqrt{2}}{2} \cdot \frac{4}{\pi} = \frac{2\sqrt{2}}{\pi}$$

$$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$Min \cdot (\pi/4) \leq \int_{\pi/4}^{\pi/2} \frac{\sin x}{x} dx \leq Max \cdot \frac{\pi}{4} \quad ; \quad Max = \frac{2\sqrt{2}}{\pi}$$

$$\Rightarrow \int_{\pi/4}^{\pi/2} \frac{\sin x}{x} dx < \frac{2\sqrt{2}}{\pi} \cdot \frac{\pi}{4}$$

$$\int_{\pi/4}^{\pi/2} \frac{\sin x}{x} dx \leq \frac{\sqrt{2}}{2} \quad V_3.$$

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Verify without evaluating the integrals.

#23)

$$\frac{\pi}{6} \leq \int_{\pi/6}^{\pi/2} \sin x \, dx \leq \pi/3$$

$f(x) = \sin x$, $\sin \pi/2 = 1$, $\sin \pi/6 = 1/2$.

$$\frac{\pi}{2} - \frac{\pi}{6} = \frac{3\pi - \pi}{6} = \frac{\pi}{3}$$

$$\Rightarrow \text{Min} \cdot \frac{\pi}{3} \leq \int_{\pi/6}^{\pi/2} \sin x \, dx \leq \text{Max} \cdot \frac{\pi}{3}$$

Min of $f(x) = 1/2$ on $[\pi/6, \pi/2]$
Max of $f(x) = 1$ on $[\pi/6, \pi/2]$

$$\Rightarrow \frac{1}{2} \cdot \frac{\pi}{3} \leq \int_{\pi/6}^{\pi/2} \sin x \, dx \leq 1 \cdot \frac{\pi}{3}$$

$$\Rightarrow \frac{\pi}{6} \leq \int_{\pi/6}^{\pi/2} \sin x \, dx \leq \pi/3 \quad \checkmark.$$

You do #24 and #26

Problems of #26 are those of section 5.5.

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#25) a) Use a Linear approximation $L(x)$ to an appropriate function $f(x)$ with an appropriate value of $a$, to estimate $\sqrt[3]{25}$.

Let $f(x) = \sqrt[3]{x}$, $a = 27 \longrightarrow f(a) = \sqrt[3]{27} = 3$.

$f'(x) = \frac{1}{3} x^{-2/3} \longrightarrow f'(27) = \frac{1}{3}(27)^{-2/3} = \frac{1}{27}$

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = 3 + \frac{1}{27}(x-27)$

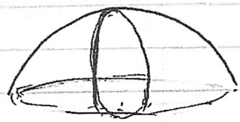
$\Rightarrow \sqrt[3]{25} \approx L(25) = 3 + \frac{1}{27}(25-27)$

$= 3 - \frac{2}{27} = \frac{3\times 27 - 2}{27} = \frac{79}{27}$

$= 2.9259$

b) Use differentials to estimate the amount of paint needed to apply a coat of 0.1 cm thick to a hemisphere with radius 100 m.

Coat of paint



$V = \frac{1}{2}\frac{4}{3}\pi R^3 = \frac{2}{3}\pi R^3$

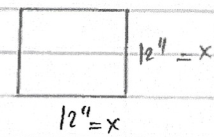
$dV = \frac{2}{3}\pi \times 3R^2\, dR$

$dV = 2\pi R^2\, dR$

$dV = 2\pi (100)^2 \times \frac{0.1}{100} = 20\pi\ m^3 = 62.83\ m^3$ of paint.

$\approx 16598$ gallons.

#25) c) The measurement of the side of a square is found to be 12 inches, with a possible error of $\frac{1}{64}$ inch. Use differentials to approximate the possible propagated error in computing the area of the square




$$A = x^2$$

$$dA = 2x\,dx \quad ; \quad dx = \pm \frac{1}{64}"$$

$$dA = 2 \times 12 \times \frac{1}{64} = \frac{24}{64} = \frac{3}{8} \text{ Inch}^2 .$$

Actually : $dA = \pm \frac{3}{8} \text{ Inch}^2$