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Section 3.5

#45) $y = \dfrac{x^2+1}{x+1}$

$$\begin{array}{r}
x-1 \\
x+1 \,\big)\, x^2+1 \\
x^2+x \\
\hline
1-x \\
-1-x \\
\hline
2
\end{array}$$

$= (x-1) + \dfrac{2}{(x+1)}$

$\lim\limits_{x \to \infty} \dfrac{x^2+1}{x+1} = (x-1) \longrightarrow$ $y = x-1$ is the oblique asymptote.

46) $y = \dfrac{2x^3+x^2+x+3}{x^2+2x}$

$$\begin{array}{r}
2x-3 \\
x^2+2x \,\big)\, 2x^3+x^2+x+3 \\
2x^3+4x^2 \\
\hline
-3x^2+x \\
-3x^2-6x \\
\hline
7x+3
\end{array}$$

$= (2x-3) + \dfrac{(7x+3)}{x^2+2x}$

$\lim\limits_{x \to \infty} y = (2x-3)$

Oblique Asymptote is $y = 2x-3$.

#48) $y = \dfrac{5x^4+x^2+x}{x^3-x^2+2}$

$$\begin{array}{r}
5x+5 \\
x^3-x^2+2 \,\big)\, 5x^4+x^2+x \\
5x^4-5x^3+10x \\
\hline
5x^3+x^2-9x \\
5x^3-5x^2+10 \\
\hline
6x^2-9x-10
\end{array}$$

$\lim\limits_{x \to \infty} \dfrac{5x^4+x^2+x}{x^3-x^2+2} = \lim\limits_{x \to \infty} \left(5x+5 + \dfrac{6x^2-9x-10}{x^3-x^2+2}\right)$

$= (5x+5) \Longrightarrow$ $y = 5x+5$ is the oblique asymptote.

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#2)

$x - y = 100 \implies y = x - 100$

Product $= x \cdot y$ to be a minimum.

$P = x(x-100)$

$P = x^2 - 100x$

$\frac{dP}{dx} = 2x - 100$

$\frac{dP}{dx} = 0 \implies x = 50$

x	$-\infty$		50		$+\infty$
dP/dx		$-$	0	$+$	
$P(x)$		$\searrow$		$\nearrow$	

$\implies x = 50,\ y = 50 \rightarrow$ Product $= -2500$

#3)

$x \cdot y = 100$

$S = x + y =$ Minimum

$\rightarrow y = \frac{100}{x}$

$S = x + \frac{100}{x}$

$\frac{dS}{dx} = 1 - \frac{100}{x^2} = \frac{x^2 - 100}{x^2}$; $x \neq 0$; $x > 0$

x		-10		10	
$\frac{dS}{dx}$	$+$	0	$-$	0	$+$
	$\nearrow$		$\searrow$		$\nearrow$

$\rightarrow \boxed{x = 10,\ y = 10}$

local Min $(10, 20)$

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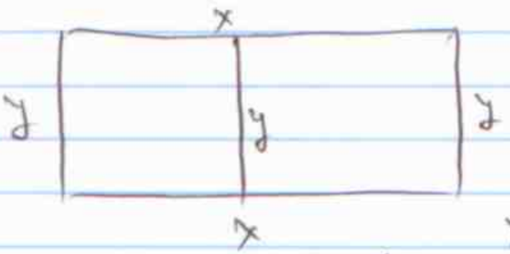
#13



$$x \cdot y = 1.5\times10^{6}\ ft^2 \longrightarrow y = \frac{1.5\times10^{6}}{x}$$

$P = 3y + 2x$ to be minimum.

$$P = \frac{3\times1.5\times10^{6}}{x} + 2x$$

$$\frac{dP}{dx} = -\frac{4.5\times10^{6}}{x^2} + 2$$

$$= \frac{-4.5\times10^{6} + 2x^2}{x^2} \quad ; x \neq 0$$

$$\frac{dP}{dx} = 0 \Longrightarrow x = +\left(\frac{4.5\times10^{6}}{2}\right)^{1/2}$$

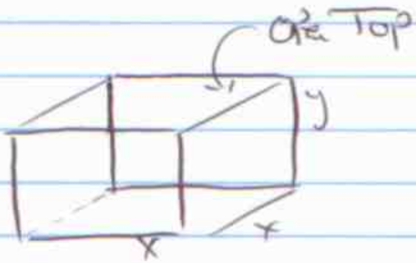
$$x = 1.5\times10^{3}\ ft$$

$$\longrightarrow y = \frac{1.5\times10^{6}}{1.5\times10^{3}} = 10^{3}\ ft.$$

$$P = 3\times10^{3} + 2\times1.5\times10^{3}$$

$$= \underline{6\times10^{3}\ ft}.$$

#14)



$$x^2 \cdot y = 32000\ cm^3$$

Surface Area $= x^2 + 4xy$

$$\Rightarrow y = \frac{32000}{x^2}$$

$$\Longrightarrow S = x^2 + 4x\cdot\frac{32000}{x^2} = x^2 + \frac{128000}{x}$$

$$\frac{dS}{dx} = 2x - \frac{128000}{x^2} \quad ; x \neq 0$$

$$\frac{dS}{dx} = 0 \longrightarrow \frac{2x^3 - 128000}{x^2} = 0$$

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$\longrightarrow \quad \dfrac{2x^3 - 128000}{x^2} = 0 \qquad ; x \neq 0$

$\dfrac{dS}{dx} = 0 \implies x = \left(\dfrac{128000}{2}\right)^{1/3} = \underline{\underline{40 \text{ cm}}}$

$\implies y = \dfrac{32000}{(40)^2} = 20 \text{ cm}$

$\longrightarrow$ Box dimensions are $40 \times 40 \times 20$ (cm).