STUDY GUIDE FOR FINAL EXAMINATION

MATH 1501

1. Evaluate each of the following limits or state it does not exist. If it does not exist, give a reason.

   (a) $\lim\limits_{x\to 1}\dfrac{x^2+7x-8}{x-1}$ (b) $\lim\limits_{x\to\infty}\dfrac{\sin x}{x^2}$

   (c) $\lim\limits_{x\to 3} f(x)$ where $f(x)=\begin{cases} x^2 & \text{if } x\le 3 \\ 9-x & \text{if } x>3 \end{cases}$ (d) $\lim\limits_{x\to -1^-}\dfrac{x^3}{(x+1)^2}$

   (e) $\lim\limits_{x\to 0}\dfrac{\sin 3x}{\frac{x}{3}}$

2. Use the ***<u>definition of the derivative</u>***, $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, to compute the derivative of $f(x)=2x^2-3x+6$.

3. If f is a differentiable function and $y=(x^2)\cdot f(x)$, find $\dfrac{dy}{dx}$.

4. Given that f and g are differentiable functions, $f(4) = 0$, $f'(4) = 8$, $g(4) = 1$, $g'(4) = 0.25$, and $h(x)=\dfrac{f(x)}{g(x)}$. Find $h'(4)$.

5. If $f(x)=\sin(x^3)$, find $f'(x)$.

6. If $f(x)=\cos[\cos(\pi x)]$, find $f'(1.1)$.

7. Find an equation of the tangent line to the graph of the equation $x^2+9xy+y^2=36$ at the point (0, 6).

8. Find the maximum value of $f(x)=x^3+2x^2-4x$ on the interval $[-3, 1]$.

9. The acceleration due to gravity on Mars is 3.72 m/sec^2. If a rock is thrown straight up from the surface of Mars with an initial velocity of 23 m/sec, how high will the rock go before it starts to fall?

10. Find the maximum value and minimum value of $f(x)=(x-3)^{2/3}$ on [0, 4].

11. Let $f(x)=\dfrac{3x+6}{x-1}$.
    (a) Find the intervals on which the graph is increasing and decreasing.
    (b) Find any local extrema.
    (c) Find the intervals on which the graph is concave up and concave down.
    (d) Find any inflection points.
    (e) Find all asymptotes.

12. Repeat problem 11 for $f(x)=x^5-5x^4$.

13. A spherical balloon is being blown up at a rate of 100 cm^3/min. At what rate is its radius r changing when r is 4 cm?

14. Given that y is defined implicitly as a function of x by the equation $x=\cos(xy)$, find $y'\left(\dfrac{\sqrt{2}}{2}\right)$. <u>HINT</u>: To find out what y is when $x=\dfrac{\sqrt{2}}{2}$, you will need to use the inverse cosine.

15. Determine if the Mean Value Theorem (for derivatives) applies to $f(x)=\sin(3x)$ on the interval $\left[0,\dfrac{\pi}{4}\right]$. If it does, find the value of c whose existence is guaranteed by the Mean Value Theorem. (Use the inverse trigonometric functions on your calculator to approximate the value(s) for c.)

16. Let R be the region bounded by $y=x^2-4x+6$, x = 1, x = 4, and y =0.
    (a) Find the area of the region.
    (b) Find the volume obtained by revolving the region about the x-axis.
    (c) Find the volume obtained by revolving the region about the y-axis.
    (d) Find the volume obtained by revolving the region about the line x = 8.

17. Compute the following antiderivatives.

(a) $\displaystyle\int \frac{x^2+1}{x^2}\,dx$

(b) $\displaystyle\int x\sqrt{x^2+4}\;dx$

(c) $\displaystyle\int x^2\cos^5(x^3+5)\sin(x^3+5)\;dx$

18. Use the Fundamental Theorem of Calculus, Part 2 (FTC2) to evaluate the following definite integrals.

(a) $\displaystyle\int_{-\pi/2}^{\pi/2}(2x+\cos x)\;dx$

(b) $\displaystyle\int_0^2 \frac{t^3}{\sqrt{t^4+9}}\;dt$

(c) $\displaystyle\int_0^{\pi/6}\sin^3\theta\,\cos\theta\;d\theta$

(d) $\displaystyle\int_0^1 \frac{x+2}{(x^2+4x+1)^2}\;dx$

19. Use the Fundamental Theorem of Calculus, Part 1 (FTC1) to find $g'(x)$.

(a) $\displaystyle g(x)=\int_0^x \sqrt[4]{1+t^4}\;dt$

(b) $\displaystyle g(x)=\int_1^{\sqrt{x}} \frac{t^2}{t^2+1}\;dt$

(c) $\displaystyle g(x)=\int_{x^2-3x}^{0}(t+\sin t)\;dt$

(d) $\displaystyle g(x)=\int_{\tan x}^{x^2}\frac{1}{\sqrt{2+t^4}}\,dt$

20. If $f(x)=3x^2\sqrt{x^3-4}$, find the average value of f on [2, 5].

21. A rectangular box is to be made by cutting out equal squares from each corner of a piece of cardboard 10 inches by 16 inches and then folding up the sides. What must be the length of the side of the square cut out if the volume is to be maximized? What is the maximum volume?

22. Find the derivative of $y=5\sin^4(x^3-3x^2)$.

23. Use the properties of integrals to verify the following inequalities without evaluating the integrals.

(a) $\displaystyle\int_0^1 x^2\cos x\;dx\le\frac{1}{3}$

(b) $\displaystyle\int_{\pi/4}^{\pi/2}\frac{\sin x}{x}\,dx\le\frac{\sqrt{2}}{2}$

(c) $\displaystyle\frac{\pi}{6}\le\int_{\pi/6}^{\pi/2}\sin x\;dx\le\frac{\pi}{3}$

24. If $\int_0^1 f(x)\,dx = 2$, $\int_1^2 f(x)\,dx = 3$, $\int_0^1 g(x)\,dx = -1$ and $\int_0^2 g(x)\,dx = 4$, use the properties of definite integrals to calculate each of the following integrals.

(a) $\int_0^2 [2f(x)+g(x)]\,dx$ (b) $\int_0^1 [2f(x)+g(x)]\,dx$

(c) $\int_2^1 [2f(x)+5g(x)]\,dx$ (d) $\int_1^1 [3f(x)+2g(x)]\,dx$

(e) $\int_0^2 [\sqrt{3}f(x)+\sqrt{2}g(x)+\pi]\,dx$

25. (a) Use a linear approximation $L(x)$ to an appropriate function $f(x)$, with an appropriate value of a, to estimate $\sqrt[3]{25}$. (See Section 3.9)
(b) Use differentials to estimate the amount of paint needed to apply a coat of paint 0.1 cm thick to a hemispherical dome with radius 100 m.
(c) The measurement of the side of a square is found to be 12 inches, with a possible error of $\frac{1}{64}$ inch. Use differentials to approximate the possible propagated error in computing the area of the square.

26. Find the average value of the given function on the indicated interval.

(a) $f(x) = 8 - x^2$; $[0, 9]$ (b) $g(t) = 8\sqrt{t}$; $[1, 36]$

(c) $f(x) = 3x\sin(x^2)$; $[0, \sqrt{\pi}\,]$

ANSWERS

1. (a) 9 (b) 0
(c) does not exist (d) $-\infty$
(e) 9

2. $f'(x) = 4x - 3$

3. $\dfrac{dy}{dx} = x^2 f'(x) + 2x\, f(x)$

4. $h'(4) = 8$

5. $f'(x) = 3x^2\cos(x^3)$

6. $f'(1.1) \approx 0.7903$

7. $y = -\dfrac{9}{2}x + 6$

8. $f(-2) = 8$

9. 71.1 m and t = 6.183 sec

10. max: $f(0) = \sqrt[3]{9}$; min: $f(3) = 0$

11. (a) dec: $(-\infty, 1), (1, \infty)$
    (b) no local extrema
    (c) concave upward: $(1, \infty)$; concave downward: $(-\infty, 1)$
    (d) no inflection points
    (e) horizontal asymptote: $y = 3$; vertical asymptote: $x = 1$

12. (a) inc: $(-\infty, 0), (4, \infty,)$; dec: $(0, 4)$
    (b) local max: $f(0) = 0$; $f(4) = -256$
    (c) concave upward: $(3, \infty)$; concave downward: $(-\infty, 3)$
    (d) $(3, -162)$
    (e) no asymptotes

13. $\dfrac{25}{16\pi} \approx 0.4974$ cm/min

14. $-2 - \dfrac{\pi}{2} \approx -3.5708$

15 MVT applies since $f(x)$ is continuous on $\left[0, \dfrac{\pi}{4}\right]$ and differentiable on $\left(0, \dfrac{\pi}{4}\right)$.
$c \approx 0.422$

16. (a) 9 (b) $\dfrac{153\pi}{5}$
    (c) $\dfrac{99\pi}{2}$ (d) $\dfrac{189\pi}{2}$

17. (a) $x - \dfrac{1}{x} + C$ (b) $\dfrac{1}{3}(x^2 + 4)^{3/2} + C$
    (c) $-\dfrac{1}{18}\cos^6(x^3 + 5) + C$

18. (a) 2 (b) 1
    (c) $\dfrac{1}{64}$ (d) $\dfrac{5}{12}$

19. (a) $g'(x) = \sqrt[4]{1 + x^4}$ (b) $g'(x) = \dfrac{x}{2\sqrt{x}(x+1)}$
    (c) $g'(x) = -(2x-3)[(x^2 - 3x) + \sin(x^2 - 3x)]$
    (d) $g'(x) = -\dfrac{\sec^2 x}{\sqrt{2 + \tan^4 x}} + \dfrac{2x}{\sqrt{2 + x^8}}$

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21. square cut-out: 2 inches by 2 inches; volume: 144 in^3

22. $(60x^2 - 120x)\cos(x^3 - 3x^2)\sin^3(x^3 - 3x^2)$

23. The answer is in working out the problems.

24. (a) 14 (b) 3
(c) -31 (d) 0
(e) $5\sqrt{3} + 4\sqrt{2} + 2\pi$

25. (a) $\sqrt[3]{25} \approx \frac{79}{27} \approx 2.9259$ (b) $20\pi \text{ m}^3 \approx 62.83 \text{ m}^3$
(c) $\pm\frac{3}{8} \text{ in}^2 = \pm 0.375 \text{ in}^2$

26. (a) -19 (b) $\frac{688}{21}$
(c) $\frac{3}{\sqrt{\pi}}$