Dr. ABDALLA 7/12/04

Section 5.1

#1) $n=5$ Rectangles $a=0$, $b=10$

$$\Delta x = \frac{b-a}{n} = \frac{10-0}{5} = 2$$

a) Lower Estimate $\rightarrow$ LEP

$$\text{Area} \approx \Delta x\left[f(0)+f(2)+f(4)+f(6)+f(8)\right]$$

$$\approx 2\left[1+3+4.4+5.4+6.4\right] = 40.4 \text{ Square Units.}$$

Upper Estimate $\rightarrow$ REP

$$\text{Area} \approx \Delta x\left[f(2)+f(4)+f(6)+f(8)+f(10)\right]$$

$$\approx 2\left[3+4.4+5.4+6.4+7\right] = 52.4 \text{ Square Units.}$$

b) $n=10$ Rectangles $\Rightarrow \Delta x = \frac{b-a}{n} = \frac{10-0}{10} = 1$

Lower Estimate $\rightarrow$ LEP

$$\text{Area} \approx \Delta x\left[f(0)+f(1)+f(2)+f(3)+f(4)+f(5)+f(6)+f(7)+f(8)+f(9)\right]$$

$$\approx 1\left[1+2.1+3+3.8+4.4+4.9+5.4+5.9+6.4+6.8\right]$$

$$\approx 43.7 \text{ Square Units}$$

Upper Estimate $\rightarrow$ REP

$$\text{Area} \approx \Delta x\left[f(1)+f(2)+f(3)+f(4)+f(5)+f(6)+f(7)+f(8)+f(9)+f(10)\right]$$

$$\approx 1\left[2.1+3+3.8+4.4+4.9+5.4+5.9+6.4+6.8+7\right]$$

$$\approx 49.7 \text{ Square Units.}$$

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#5) Estimate the area under the graph of $f(x) = 1+x^2$ from $x=-1$ to $x=2$ using 3 rectangles and REP. Then improve your estimate by using six rectangles.

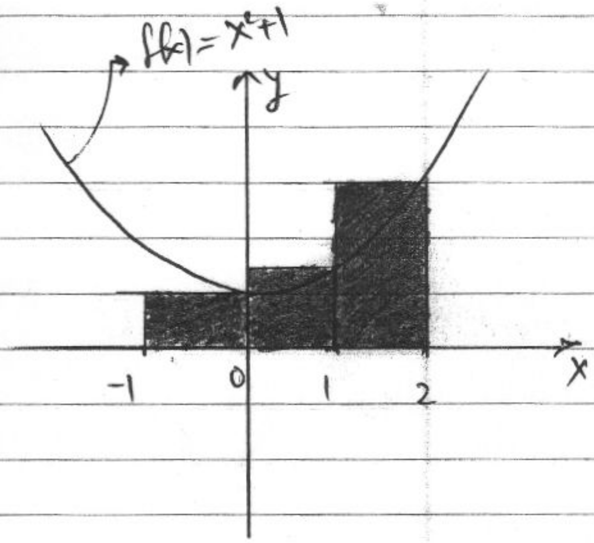
$n=3$ Rectangles.

$$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{3} = \frac{3}{3} = 1$$

REP

$$\text{Area} \approx \Delta x\left[f(0)+f(1)+f(2)\right]$$

$$\approx 1\left[1+2+5\right] = 8 \text{ Square Units.}$$



Now use $n=6$ Rectangles.

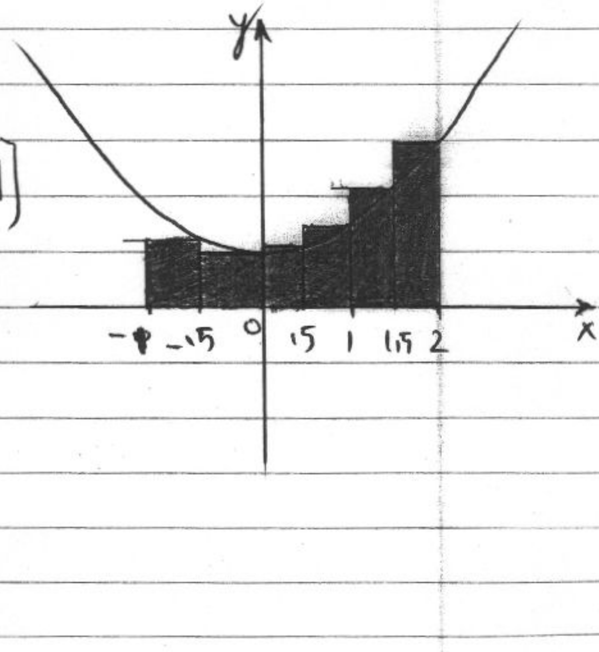
$$\Delta x = \frac{2-(-1)}{6} = \frac{3}{6} = 1/2$$

REP

$$\text{Area} = \Delta x\left[f(-.5)+f(0)+f(.5)+f(1)+f(1.5)+f(2)\right]$$

$$\approx \frac{1}{2}\left[1.25+1+1.25+2+3.25+5\right]$$

$$\cong 6.875 \text{ Square Units.}$$



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#5) Continue;

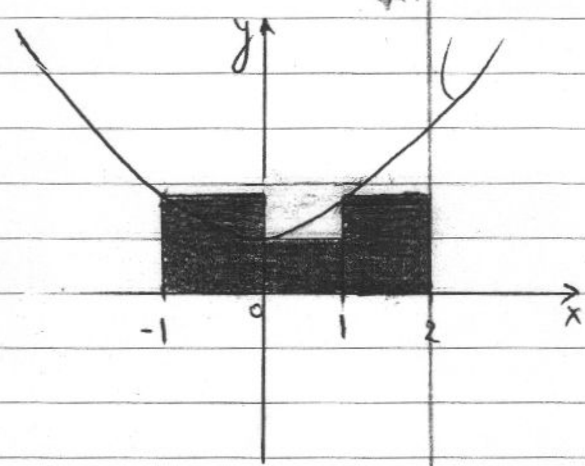
Now were going to repeat the problem but at the LEP of the base

$\Delta x = \frac{b-a}{n}$, $n = 3 \Longrightarrow \Delta x = \frac{2-(-1)}{3} = 1$

$f(x) = x^2+1$

$Area \approx \Delta x\left[f(-1) + f(0) + f(1)\right]$

$\approx 1\left[2+1+2\right] = 5$ Square Units

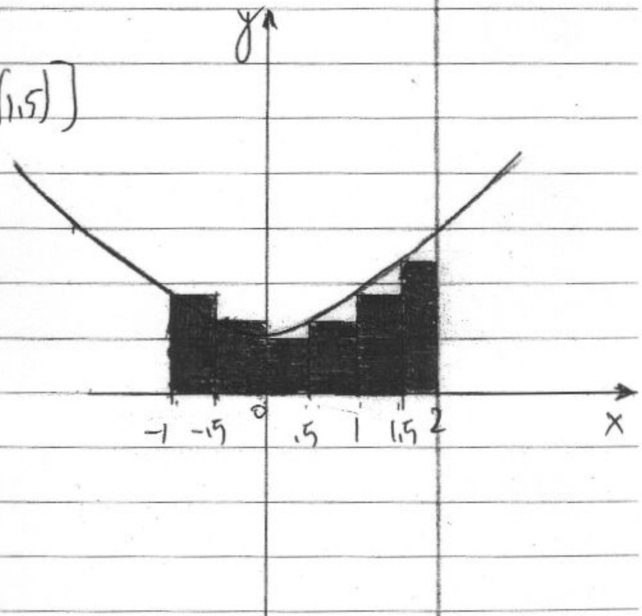


$n = 6 \Longrightarrow \Delta x = \frac{b-a}{n} = \frac{2-(-1)}{6} = \frac{3}{6} = 0.5$

$Area = \Delta x\left[f(-1) + f(-.5) + f(0) + f(.5) + f(1) + f(1.5)\right]$

$= \frac{1}{2}\left[2 + 1.25 + 1 + 1.25 + 2 + 3.25\right]$

$Area \approx 5.375$ Square Units.



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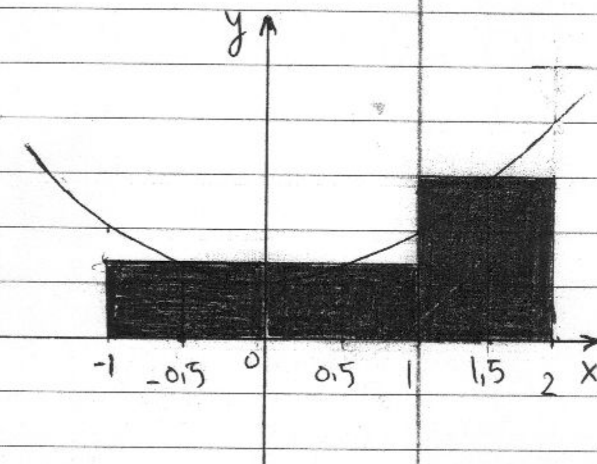
Section 5.1

#5) Continue

Now we're going to repeat the problem but at the Middle Point of the base

$a = -1, \; b = 2$

$n = 3 \implies \Delta x = \frac{b-a}{3} = \frac{2+1}{3} = 1$



Area $\cong 1\left[f(-0.5) + f(0.5) + f(1.5)\right]$

$\approx 1\left[1.25 + 1.25 + 3.25\right]$

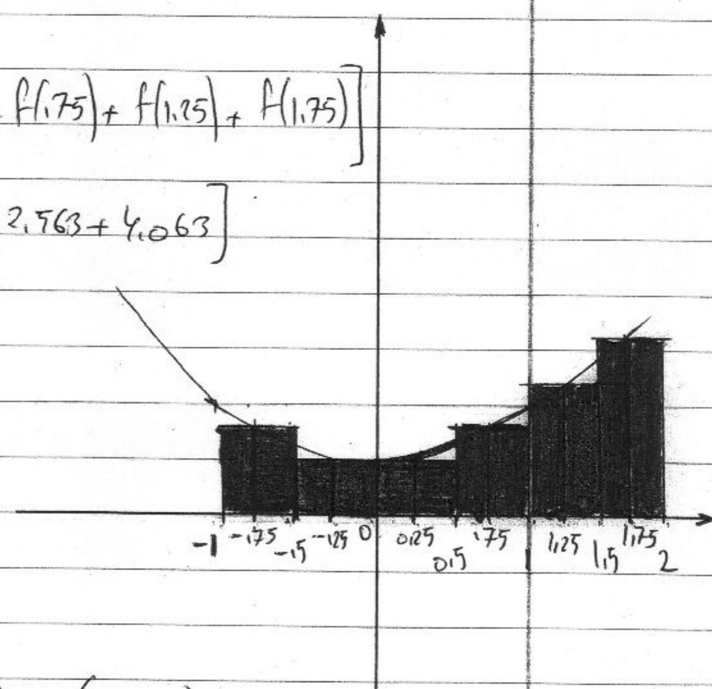
≈ 5.75 Square Units

$n = 6 \implies \Delta x = \frac{2+1}{6} = \frac{1}{2}$

Area $\approx \Delta x\left[f(-.75) + f(-.25) + f(.25) + f(.75) + f(1.25) + f(1.75)\right]$

$\approx 0.5\left[1.563 + 1.063 + 1.063 + 1.563 + 2.563 + 4.063\right]$

$\approx \approx 5.93$ Square Units.



Exact Area $= \int_{-1}^{2} (1+x^2)\,dx$

$= \left(x + \frac{x^3}{3}\right)\Big|_{-1}^{2} = \left(2 + \frac{8}{3}\right) - \left(-1 - \frac{1}{3}\right)$

$= 6$ Square Units.