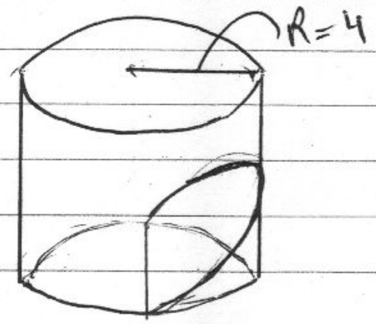
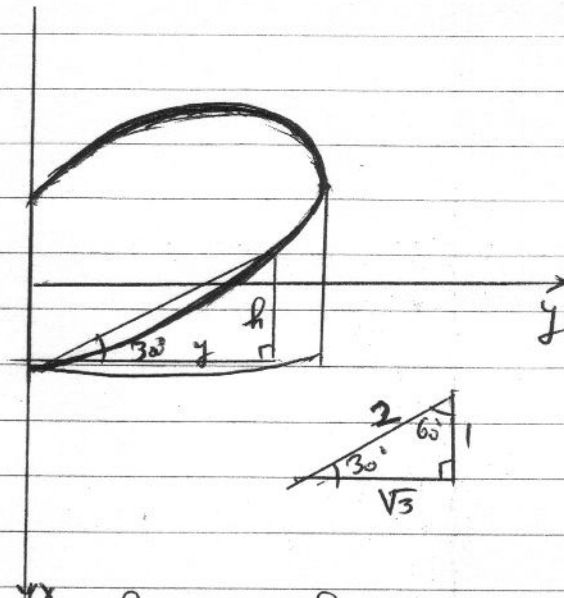
Dr. RABDAWI

CLASSICS

I)





$$\text{Volume} = \int_{-4}^{4} A(x)\,dx$$

$$A(x) = \frac{y h}{2} \quad , \quad \tan 30° = \frac{1}{\sqrt{3}} = \frac{h}{y} \implies h = \frac{y}{\sqrt{3}}$$

$$A(x) = \frac{y}{2} \cdot \frac{y}{\sqrt{3}} = \frac{y^2}{2\sqrt{3}} \quad ; \text{ but } \quad x^2 + y^2 = 16$$

$$y^2 = 16 - x^2$$

$$\implies A(x) = \frac{16 - x^2}{2\sqrt{3}}$$

$$\text{Volume} = \int_{-4}^{4} \left(\frac{16-x^2}{2\sqrt{3}}\right) dx = 2\int_{0}^{4} \frac{(16-x^2)}{2\sqrt{3}}\,dx$$

$$\text{Volume} = \frac{1}{\sqrt{3}} \int_{0}^{4} (16 - x^2)\,dx = \frac{1}{\sqrt{3}} \left(16x - \frac{x^3}{3}\right)\Big|_{0}^{4}$$

$$\text{Volume} = \frac{1}{\sqrt{3}} \left[\left(64 - \frac{64}{3}\right) - (0 - 0)\right]$$

$$= \frac{1}{\sqrt{3}} \cdot 2 \times \frac{64}{3} = \frac{128}{3\sqrt{3}} = \frac{128\sqrt{3}}{9}$$

Cubic Units.

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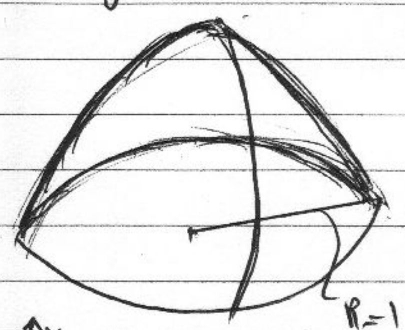
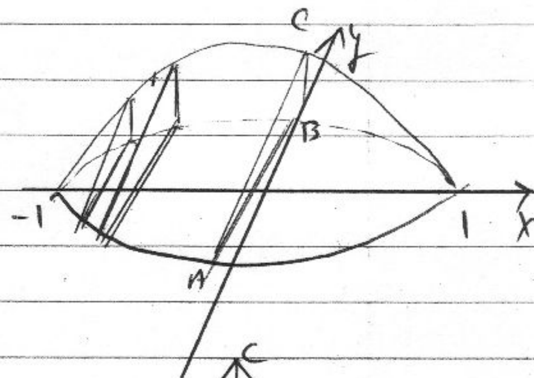
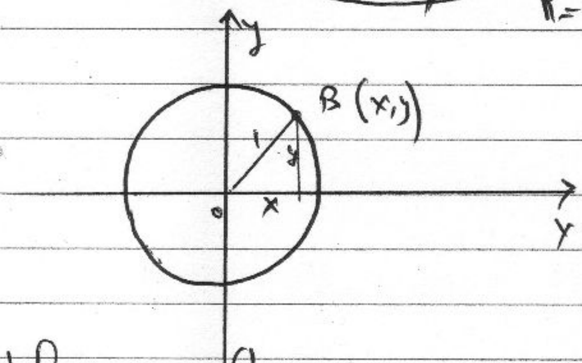
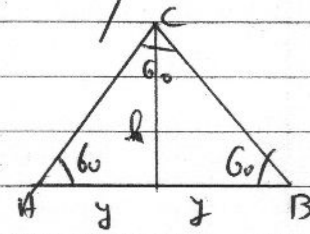
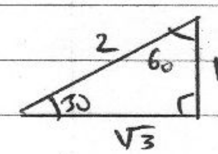
II) Volume of a loaf of bread with a circular base and an equilateral triangle as a cross section.











$$\text{Volume} = \int_{-1}^{1} A(x)\,dx \quad , \quad A = 2y\frac{h}{2} = yh$$

$$\tan 60° = \frac{h}{y} = \sqrt{3} \longrightarrow h = y\sqrt{3}$$

$$\rightarrow A(x) = y \cdot y\sqrt{3} = y^2\sqrt{3} \quad ; \text{ But } x^2 + y^2 = 1^2 = 1$$

$$\longrightarrow y^2 = 1 - x^2$$

$$\therefore A(x) = \sqrt{3}\,(1 - x^2)$$

$$\text{Volume} = \int_{-1}^{1} \sqrt{3}(1-x^2)\,dx = 2\sqrt{3}\int_0^1 (1-x^2)\,dx = 2\sqrt{3}\left(x - \frac{x^3}{3}\right)\Big|_0^1$$

$$= 2\sqrt{3}\left[\left(1-\frac{1}{3}\right) - (0-0)\right] = \frac{4\sqrt{3}}{3}$$

Cubic Units

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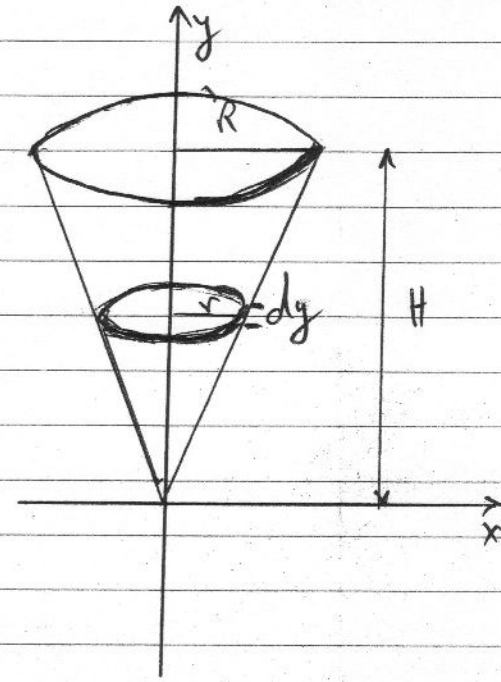

III) Volume of a Right Circular Cone

Volume = $\frac{1}{3}$ Area of Base × Height

Proof:

$$\text{Volume} = \int_0^H A(y)\, dy$$

$$A(y) = \pi r^2 \quad ; \quad \frac{R}{r} = \frac{H}{y}$$

$$\Rightarrow r = \frac{yR}{H}$$

$$\Rightarrow A(y) = \pi \frac{y^2 R^2}{H^2} = \frac{\pi R^2}{H^2} y^2$$

$$\therefore \text{Volume} = \int_0^H \frac{\pi R^2}{H^2} y^2\, dy = \frac{\pi R^2}{H^2} \frac{y^3}{3} \Big|_0^H$$

$$= \frac{\pi R^2}{H^2} \left(\frac{H^3}{3} - 0 \right)$$

$$= \frac{1}{3} \pi R^2 H$$

Volume = $\frac{1}{3}$ Area of Base × Height.

QED

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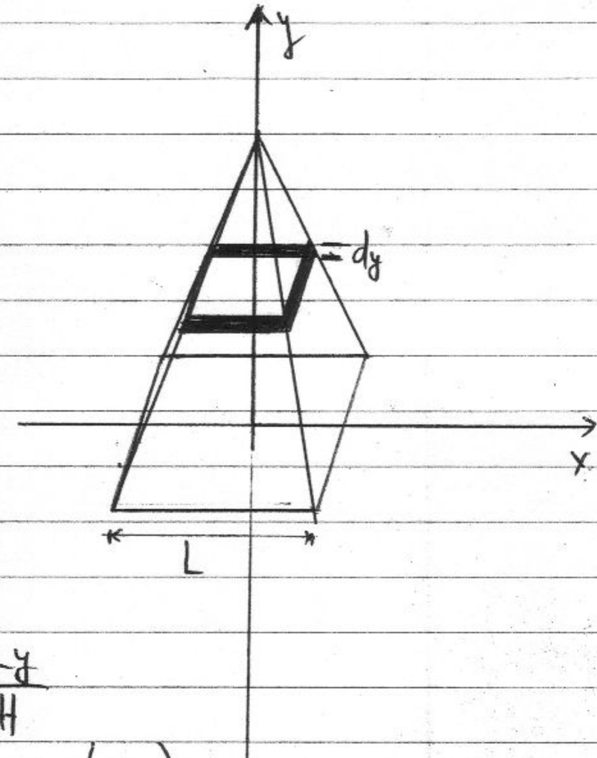
IV) Volume of a square base pyramid.

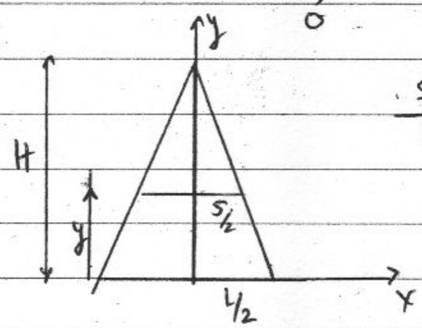
Volume $= \frac{1}{3}$ Area of the Base $\times$ Height

$$= \frac{1}{3} S^2 \times H.$$



Proof:

$$\text{Volume} = \int_0^H A(y)\,dy = \int_0^H s^2\,dy$$



$$\frac{s/2}{L/2} = \frac{H-y}{H}$$

$$\Longrightarrow \frac{s}{L} = \frac{H-y}{H}$$

$$\Longrightarrow s = \frac{L}{H}(H-y)$$

$$\Longrightarrow \text{Volume} = \int_0^H \frac{L^2}{H^2}(H-y)^2\,dy = \frac{L^2}{H^2}\int_0^H (H^2 - 2yH + y^2)\,dy$$

$$\text{Volume} = \frac{L^2}{H^2}\left(H^2 y - \frac{2y^2 H}{2} + \frac{y^3}{3}\right)\Bigg|_0^H$$

$$= \frac{L^2}{H^2}\left[\left(H^3 - H^3 + \frac{H^3}{3}\right) - (0 - 0 + 0)\right]$$

$$= \frac{L^2 H}{3} = \frac{1}{3} L^2 \times H$$

$$= \frac{1}{3} \text{ Area of Base} \times \text{Height}$$

QED

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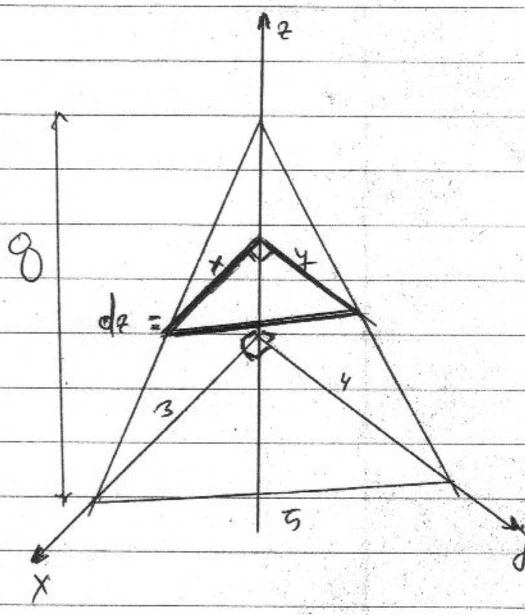
V) Volume of a pyramid with a triangular cross section.

Volume = $\frac{1}{3}$ Area of Base × Height



Proof:

$$\frac{8}{3} = \frac{8-z}{x} \Rightarrow x = \frac{3}{8}(8-z)$$

$$\frac{8}{4} = \frac{8-z}{y} \Rightarrow y = \frac{8-z}{2}$$

$$\text{Volume} = \int_0^8 A(z)\,dz$$

$$A(z) = \frac{xy}{2} = \frac{1}{2}\,\frac{3}{8}(8-z)\cdot\left(\frac{8-z}{2}\right)$$

$$= \frac{3}{32}(8-z)^2$$

$$\text{Volume} = \int_0^8 \frac{3}{32}(8-z)^2\,dz$$

let $u = 8-z \Rightarrow du = -dz \Rightarrow dz = -du$

$$\text{Volume} = -\frac{3}{32}\int_8^0 u^2\,du = \frac{3}{32}\int_0^8 u^2\,du$$

$$= \frac{3}{32}\,\frac{u^3}{3}\Big|_0^8 = \frac{1}{32}\left(8^3 - 0\right)$$

$$= \frac{1}{32}\times 64\cdot 8 = 2\times 8 = 16 \text{ Cubic Units}$$

$$= \frac{1}{3}\left(\frac{3\cdot 4}{2}\right)\cdot 8 = 16 \text{ Cubic Units}$$

$$= \frac{1}{3}(\text{Area of Base})\cdot\text{Height}$$

QED