

Solution to #27 Section 1.1

$$(x^2 + y^2) dx + (x^2 - xy) dy = 0, \quad G(x+y)^2 = x e^{y/x}$$

$$\Rightarrow G = \frac{x e^{y/x}}{(x+y)^2}$$

$$(x^2 - xy) dy = -(x^2 + y^2) dx$$

$$\frac{dy}{dx} = \frac{(x^2 + y^2)}{(xy - x^2)} \quad \text{To be verified.}$$

Now we have  $G(x+y)^2 = x e^{y/x}$

$$\Rightarrow 2G(x+y)(1+y') = 1 e^{y/x} + \left(\frac{y'x - 1 \cdot y}{x^2}\right) e^{y/x} \cdot x$$

$$2G(x+y) + 2G(x+y)y' = e^{y/x} + \frac{(y'x - y) e^{y/x}}{x}$$

$$x \neq 0, \quad 2G(x^2 + xy) + 2G(x^2 + xy)y' = x e^{y/x} + y' x e^{y/x} - y e^{y/x}$$

$$y' [2G(x^2 + xy) - x e^{y/x}] = (x - y) e^{y/x} - 2G(x^2 + xy)$$

$$y' = \frac{(x - y) e^{y/x} - 2G(x^2 + xy)}{2G(x^2 + xy) - x e^{y/x}}$$

$$= \frac{(x - y) e^{y/x} - 2 x e^{y/x} \frac{(x^2 + xy)}{(x+y)^2}}{2 \frac{x e^{y/x} (x^2 + xy)}{(x+y)^2} - x e^{y/x}}$$

$$= \frac{(x - y) e^{y/x} - 2 x^2 \frac{(x^2 + xy)}{(x+y)^2}}{\frac{2 x e^{y/x} (x^2 + xy)}{(x+y)^2} - x e^{y/x}}$$

$$= \frac{(x - y) - 2 x^2 / (x+y)}{\frac{2 x^2}{(x+y)} - x} = \frac{(x - y)(x+y) - 2 x^2}{2 x^2 - x(x+y)}$$

$$= - \frac{(x^2 + y^2)}{x^2 - xy} \quad \checkmark$$