Dr. ZABDAWI 4/11/05

Section 7.1

#5) $f(t) = \begin{cases} \sin t , & 0 < t < \pi \\ 0 , & t \geq \pi \end{cases}$

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\,dt = \int_0^\pi e^{-st} \sin t \, dt$$

let $u = \sin t \Rightarrow du = \cos t\,dt$

$dV = e^{-st} dt \Longrightarrow V = \frac{e^{-st}}{-s}$

$$\int_0^\pi e^{-st} \sin t \, dt = -\frac{e^{-st}}{s} \sin t \Big|_0^\pi + \frac{1}{s} \int_0^\pi e^{-st} \cos t \, dt$$

$$= -\frac{1}{s}(0 - 0) + \frac{1}{s} \int_0^\pi e^{-st} \cos t \, dt$$

let $u = \cos t \Longrightarrow du = -\sin t \, dt$

$dV = e^{-st} dt \Longrightarrow V = -\frac{e^{-st}}{s}$

$$\int_0^\pi e^{-st} \sin t \, dt = \frac{1}{s}\left[-\frac{e^{-st}}{s} \cos t \Big|_0^\pi - \frac{1}{s} \int_0^\pi e^{-st} \sin t \, dt \right]$$

$$\left(\int_0^\pi e^{-st} \sin t \, dt \right)\left(1 + \frac{1}{s^2}\right) = -\frac{1}{s^2}\left[-e^{-s\pi} - 1 \right] = \frac{e^{-s\pi} + 1}{s^2}$$

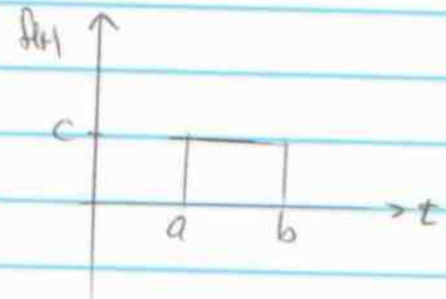
$$\therefore \int_0^\pi e^{-st} \sin t \, dt = \frac{e^{-s\pi} + 1}{s^2} \cdot \frac{s^2}{s^2 + 1}$$

$$= \frac{e^{-s\pi} + 1}{(s^2 + 1)} \quad ; s > 0$$

Section 7.1

4/10/05

#10) $f(t) =$



$$f(t) = c\left[u(t-a) - u(t-b)\right] \quad ; \text{use } \mathcal{L}\left(f(t-a)\,u(t-a)\right) = e^{-as}F(s)$$

$$\mathcal{L}\left(f(t)\right) = c\left[\frac{e^{-as}}{s} - \frac{e^{-bs}}{s}\right] = \frac{c}{s}\left[e^{-as} - e^{-bs}\right].$$