Section 7.5 Dr. ZABDAWI

#1(a) $y'' - 2y' + 5y = 1 + t$, $y(0) = 0$, $y'(0) = 4$

$$s^2 Y(s) - s\cancel{y(0)} - y'(0) - 2\left[sY(s) - \cancel{y(0)}\right] + 5Y(s) = \frac{1}{s} + \frac{1}{s^2}$$

$$Y(s)\left[s^2 - 2s + 5\right] = 4 + \frac{1}{s} + \frac{1}{s^2} = \frac{4s^2 + s + 1}{s^2}$$

$$Y(s) = \frac{4s^2 + s + 1}{s^2(s^2 - 2s + 5)}$$

Note that $s^2 - 2s + 5$ is irreducible because $b^2 - 4ac < 0$

i.e. $(-2)^2 - 4 \cdot 1 \cdot 5 = -16 < 0$

$$\frac{4s^2 + s + 1}{s^2(s^2 - 2s + 5)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs + D}{(s^2 - 2s + 5)}$$

$$\Rightarrow As(s^2 - 2s + 5) + B(s^2 - 2s + 5) + (Cs + D)s^2 = 4s^2 + s + 1$$

$$As^3 - 2As^2 + 5As + Bs^2 - 2Bs + 5B + Cs^3 + Ds^2 = 4s^2 + s + 1$$

$$(A + C)s^3 + (-2A + B + D)s^2 + (5A - 2B)s + 5B = 4s^2 + s + 1$$

$$5B = 1 \Rightarrow B = 1/5 \quad \text{OR} \quad B = \left.\frac{4s^2 + s + 1}{s^2 - 2s + 5}\right|_{s=0} = 1/5$$

$$A + C = 0 \Rightarrow A = -C$$

$$5A - 2B = 1 \Rightarrow A = \frac{1}{5}(1 + 2B)$$

$$A = \frac{1}{5}\left(1 + \frac{2}{5}\right) = 7/25$$

$$\Rightarrow C = -7/25$$

$$-2A + B + D = 4$$

$$D = 2A - B + 4 = \frac{14}{25} - \frac{1}{5} + 4 = \frac{14 - 5 + 100}{25}$$

$$D = \frac{109}{25}$$

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$$Y(s) = \frac{7/25}{s} + \frac{1/5}{s^2} + \frac{-7/25\, s + 109/25}{s^2-2s+5} \quad ; \; 109 = 7+102$$

$$= \frac{7}{25}\cdot\frac{1}{s} + \frac{1}{5}\cdot\frac{1}{s^2} + \frac{-7/25(s-1) - 7/25 + 109/25}{(s-1)^2+4}$$

$$= \frac{7}{25}\cdot\frac{1}{s} + \frac{1}{5}\cdot\frac{1}{s^2} - \frac{7}{25}\frac{(s-1)}{(s-1)^2+4} + \frac{102/25}{(s-1)^2+4}$$

$$= \frac{7}{25}\cdot\frac{1}{s} + \frac{1}{5}\cdot\frac{1}{s^2} - \frac{7}{25}\frac{(s-1)}{(s-1)^2+4} + \frac{51}{25}\cdot\frac{2}{(s-1)^2+4}$$

$$\therefore Y(t) = \frac{7}{25} + \frac{1}{5}t - \frac{7}{25}e^{t}\cos 2t + \frac{51}{25}\sin 2t\, e^{t}$$

#60) $tY'' + 2tY' + 2Y = 0 \quad , \; Y(0) = 0$

$$-\frac{d}{ds}\left(s^2 Y(s) - \cancel{sY(0)} - Y'(0)\right] - 2\frac{d}{ds}\left[sY(s) - \cancel{Y(0)}\right] + 2Y(s) = 0$$

$$-2sY(s) - s^2\frac{d}{ds}(Y(s)) - 2Y(s) - 2s\frac{d}{ds}Y(s) + 2Y(s) = 0$$

$$\frac{dY}{ds}\left[-s^2 - 2s\right] = 2sY(s)$$

$$\frac{dY}{ds} = \frac{2s}{-s(s+2)}Y(s) \quad ; \; s \neq 0, \; s > 0$$

$$\frac{dY}{Y} = -\frac{2}{(s+2)}\,ds$$

$$\ln(Y(s)) = -2\ln(s+2) + c \quad = -\ln(s+2)^2 + c$$

$$\ln\left[Y(s)\times(s+2)^2\right] = c$$

$$Y(s)\times(s+2)^2 = e^{c} = c_1 \quad , \; e^{c} \text{ is a Constant}$$

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$$Y(s) \cdot (s+2)^2 = c_1$$

$$Y(s) = \frac{c_1}{(s+2)^2}$$

$$\Longrightarrow Y(t) = c_1 t e^{-2t}$$

Section 8.1

Dr. ZABDAWI

#3)

$$\frac{dx}{dt} = -y + t$$

$$\frac{dy}{dt} = x - t$$

$\Rightarrow$ $Dx + y = t$ ——— (1)

$-x + Dy = -t$ ——— (2)

D. eq (2) and add eq(1) + eq.(2)

$$y + D^2 y = t - 1$$

$\Rightarrow$ $y'' + y = 0$ (Homogeneous Part)

$r^2 + 1 = 0 \Rightarrow r = \pm i$

$y(t) = c_1 \cos t + c_2 \sin t$

$y_p = At + B$, $y_p' = A$, $y_p'' = 0$

$At + B = t - 1 \Rightarrow A = 1, B = -1$

$\therefore y(t)_{\text{Complete}} = c_1 \cos t + c_2 \sin t + t - 1$

eq(2) $\Rightarrow$ $x(t) = \frac{dy}{dt} + t$

$$\boxed{\begin{aligned} x(t) &= -c_1 \sin t + c_2 \cos t + 1 + t \\ y(t) &= c_1 \cos t + c_2 \sin t + t - 1 \end{aligned}}$$

Section 8.1 Dr. BABDAWI

#15) $(D-1)x + (D^2+1)y = 1$ —— (1)

$(D^2-1)x + (D+1)y = 2$ —— (2)

Eliminate x

$-(D+1)\,eq(1) + eq(2)$:

$-(D+1)(D^2+1)y + (D+1)y = -(D+1)1 + 2$

$-(D^3+D+D^2+1)y + (D+1)y = -1+2 = 1$

$-(y''' + y' + y'' + y) + y' + y = 1$

$-y''' - y'' = 1$

$y''' + y'' = -1$; $y(t) = y_H + y_p$

$r^3 + r^2 = 0$

$r^2(r+1) = 0 \Longrightarrow r = 0,\ r = -1$

$y_H = C_1 + C_2 t + C_3 e^{-t}$

$y_p = At^2$, $y_p' = 2At$, $y_p'' = 2A$, $y_p''' = 0$

$2A = -1 \longrightarrow A = -1/2$

$\therefore y(t) = C_1 + C_2 t + C_3 e^{-t} - \frac{1}{2}t^2$ —— (3).

Let's now eliminate y and solve for x.

$-(D+1)\,eq(1) + (D^2+1)\,eq(2)$

Continue #15 Dr. ZOBIDAWI

$$-(D+1)(D+1)x+(D^2+1)(D^2-1)x=-(D+1)1+(D^2+1)2$$

$$-(D^2-1)x+(D^4-1)x=-1+2=1$$

$$-(x''-x)+(x^{iv}-x)=1$$

$$x^{iv}-x''=1 \quad ; \; x(t)=x_H+x_p$$

$$r^4-r^2=0$$

$$r^2(r^2-1)=0 \implies r=0,\pm 1$$

$$x_H(t)=C_4+C_5t+C_6e^{-t}+C_7e^{t}$$

$$x_p=At^2, \quad x_p'=2At, \quad x_p''=2A, \quad x_p'''=0, \quad x_p^{iv}=0$$

$$-2A=1 \implies A=-1/2$$

$$\therefore x(t)=C_4+C_5t+C_6e^{-t}+C_7e^{t}-\tfrac{1}{2}t^2$$

$$y(t)=C_1+C_2t+C_3e^{t}-\tfrac{1}{2}t^2$$

We should only have 4 coefficients

$\implies C_1, C_2, C_3$ should put in terms (C_4, C_5, C_6, C_7)

Substituting back into eq. (1) we have:

$$(D-1)x+(D^2+1)y=1$$

$$x'-x+y''+y=1 \qquad \forall t, \; t>0$$

$$C_5-C_6e^{-t}+C_7e^{t}-t-\left[C_4+C_5t+C_6e^{-t}+C_7e^{t}-\tfrac{1}{2}t^2\right]+y''+C_1+C_2t+C_3e^{-t}-\tfrac{1}{2}t^2=1$$

$\forall t>0$

Continue #15 Dr. RABIDAUX(?)

$$y'(t) = C_2 - C_3e^{-t} - t$$

$$y''(t) = C_3e^{-t} - 1$$

$$\Rightarrow \left\{ C_5 - C_6e^{-t} + C_7e^{t} - t - \left[C_4 + C_5t + C_6e^{-t} + C_7e^{t} - \tfrac{1}{2}t^2\right] + C_3e^{-t} - 1 + C_1 + C_2t + C_3e^{-t} - \tfrac{1}{2}t^2 \right\} = 1 \quad \forall t > 0$$

$$\left\{ C_5 - C_6e^{-t} + \cancel{C_7e^{t}} - t - C_4 - C_5t - C_6e^{-t} - \cancel{C_7e^{t}} + \cancel{\tfrac{1}{2}t^2} + C_3e^{-t} - 1 + C_1 + C_2t + C_3e^{-t} - \cancel{\tfrac{1}{2}t^2} \right\} = 1 \quad \forall t > 0$$

$$C_5 - C_6e^{-t} - t - C_4 - C_5t - C_6e^{-t} + C_3e^{-t} - 1 + C_1 + C_2t + C_3e^{-t} = 1 \quad \forall t > 0$$

$$(C_5 - C_4 - 1 + C_1) = 1$$

$$\boxed{C_1 = 2 + C_4 - C_5}$$

$$(-1 - C_5 + C_2)t = 0 \quad \forall t > 0 \Rightarrow -1 - C_5 + C_2 = 0$$

$$\boxed{C_2 = 1 + C_5}$$

$$(-C_6 - C_6 + C_3 + C_3)e^{-t} = 0 \quad \forall t > 0 \Rightarrow \boxed{C_3 = C_6}$$

$$\therefore \quad x(t) = C_4 + C_5t + C_6e^{-t} + C_7e^{t} - \tfrac{1}{2}t^2$$

$$y(t) = (2 + C_4 - C_5) + (1 + C_5)t + C_6e^{-t} - \tfrac{1}{2}t^2$$