Dr. ZABDAWI

Section 5.2

#1) $\frac{1}{16}x'' + 2x' + x = 0$, $x(0) = 0$, $x'(0) = -1.5$

Mass = 2 Lbm , Damping force = $2\frac{dx}{dt}$, $k = 1\frac{lb}{ft.}$

$\frac{1}{16}r^2 + 2r + 1 = 0$

$r^2 + 32r + 16 = 0$

$r = -16 \pm \sqrt{16^2 - 16} = -16 \pm 4\sqrt{15}$

$\therefore X_H(t) = C_1 e^{(-16-4\sqrt{15})t} + C_2 e^{(-16+4\sqrt{15})t}$

$X'(t) = (-16-4\sqrt{15})C_1 e^{(-16-4\sqrt{15})t} + (-16+4\sqrt{15})C_2 e^{(-16+4\sqrt{15})t}$

$x(0) = 0 \implies C_1 + C_2 = 0$

$x'(0) = -1.5 \implies (-16-4\sqrt{15})C_1 + (-16+4\sqrt{15})C_2 = -1.5$

$$C_1 = \frac{\begin{vmatrix} 0 & 1 \\ -1.5 & -16+4\sqrt{15} \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ -16-4\sqrt{15} & -16+4\sqrt{15} \end{vmatrix}} = \frac{1.5}{-16+4\sqrt{15}+16+4\sqrt{15}} = \frac{1.5}{8\sqrt{15}} = \frac{\sqrt{15}}{80}$$

$$C_2 = \frac{\begin{vmatrix} 1 & 0 \\ -16-4\sqrt{15} & -1.5 \end{vmatrix}}{8\sqrt{15}} = \frac{-1.5}{8\sqrt{15}} = -\frac{\sqrt{15}}{80}$$

$\therefore X_H(t) = \frac{\sqrt{15}}{80} e^{(-16-4\sqrt{15})t} + \frac{\sqrt{15}}{80} e^{(-16+4\sqrt{15})t}$

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#2) $\frac{16}{32} x'' + x' + 2x = 0 \quad ; x(0) = -2 \ , \ x'(0) = 1$

Mass = 16 Lbm

Damping force = $\frac{dx}{dt}$

$k = 2 \frac{\text{Lbf}}{\text{ft}}$

$\frac{1}{2} r^2 + r + 2 = 0$

$r^2 + 2r + 4 = 0$

$r = -1 \pm \sqrt{1-4} = -1 \pm \sqrt{3}\, i$

$x(t) = e^{-t}\left[C_1 \cos(\sqrt{3}t) + C_2 \sin(\sqrt{3}t)\right]$

$x'(t) = -e^{-t}\left[C_1 \cos(\sqrt{3}t) + C_2 \sin(\sqrt{3}t)\right] + e^{-t}\left[-\sqrt{3}C_1 \sin\sqrt{3}t + \sqrt{3}C_2 \cos\sqrt{3}t\right]$

$x(0) = -2 \implies \boxed{C_1 = -2}$

$x'(0) = 1 \implies -\left[C_1 + 0\right] + 1\left[0 + \sqrt{3}C_2\right] = 1$

$2 + \sqrt{3}C_2 = 1$

$C_2 = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\therefore x_{11}(t) = e^{-t}\left[-2\cos(\sqrt{3}t) - \frac{\sqrt{3}}{3}\sin(\sqrt{3}t)\right]$.

Dr. [illegible]

Section 5.2

*8) $k = \frac{8}{4} = 2 \frac{lbf}{ft}$; Damping force $= \sqrt{2}\frac{dx}{dt}$

mass $= 8$ lbm

$\frac{8}{32}\frac{d^2x}{dt^2} + \sqrt{2}\frac{dx}{dt} + 2x = 0$; $x(0) = 0$, $x'(0) = 5$ fps.

$\frac{1}{4}\frac{d^2x}{dt^2} + \sqrt{2}\frac{dx}{dt} + 2x = 0$

$\frac{d^2x}{dt^2} + 4\sqrt{2}\frac{dx}{dt} + 8x = 0$

$r^2 + 4\sqrt{2}r + 8 = 0$

$r = -2\sqrt{2} \pm \sqrt{8-8} = -2\sqrt{2}$

$x_h(t) = C_1 e^{-2\sqrt{2}t} + C_2 t e^{-2\sqrt{2}t}$

$x(0) = 0 \implies \boxed{C_1 = 0} \implies x_h(t) = C_2 t e^{-2\sqrt{2}t}$

$x'(t) = C_2 e^{-2\sqrt{2}t} - 2\sqrt{2}C_2 t e^{-2\sqrt{2}t}$

$x'(0) = 5 \implies \boxed{C_2 = 5}$

$\therefore x_h(t) = 5te^{-2\sqrt{2}t}$

$x'(t) = 5e^{-2\sqrt{2}t} - 10\sqrt{2}te^{-2\sqrt{2}t}$

$x'(t) = 0 \implies t = \frac{5}{10\sqrt{2}} = \frac{\sqrt{2}}{4}$ seconds.

Maximum Displacement $= x(t)\Big|_{t=\frac{\sqrt{2}}{4}} = \frac{5\sqrt{2}}{4}e^{-2\sqrt{2}\cdot\frac{\sqrt{2}}{4}} = \frac{5\sqrt{2}}{4e}$

$= .65$ ft

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Section 5.3

#1) $m = 16\,\text{Lbm}$, $k = \frac{16}{8/3} = 6$, $x'(0) = 9$ $x(0) = 2$

Damping force $= \frac{1}{2}\frac{dx}{dt}$, $f(t) = 10\cos 3t$

$$\frac{16}{32}\frac{d^2x}{dt^2} + \frac{1}{2}\frac{dx}{dt} + 6x = 10\cos 3t$$

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + 12x = 20\cos 3t \quad \text{—— (1)}$$

$$r^2 + r + 12 = 0$$

$$r = \frac{-1 \pm \sqrt{1-48}}{2} = -\frac{1}{2} \pm \frac{\sqrt{47}}{2} i$$

$$X_H(t) = e^{-t/2}\left[c_1 \cos \frac{\sqrt{47}}{2}t + c_2 \sin\left(\frac{\sqrt{47}}{2}t\right)\right]$$

$$Y_p = A\cos 3t + B\sin 3t$$

$$Y_p' = -3A\sin 3t + 3B\cos 3t$$

$$Y_p'' = -9A\cos 3t - 9B\sin 3t$$

Substitute back into (1)

$$-9A\cos 3t - 9B\sin 3t + 3B\cos 3t - 3A\sin 3t + 12\left(A\cos 3t + B\sin 3t\right) = 20\cos 3t$$

$$\left(-9A + 3B + 12A\right)\cos 3t + \left(-9B - 3A + 12B\right)\sin 3t = 20\cos 3t$$

$$+3A + 3B = 20$$
$$-3A + 3B = 0$$

$$6B = 20$$

$$\Rightarrow B = \frac{20}{6} = \frac{10}{3} \quad , \quad A = B = \frac{10}{3}$$

$$\therefore Y_p = \frac{10}{3}\cos 3t + \frac{10}{3}\sin 3t$$

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$$X_{complete}(t) = e^{-t/2}\left[C_1\cos\left(\frac{\sqrt{47}}{2}t\right) + C_2\sin\left(\frac{\sqrt{47}}{2}t\right)\right] + \frac{10}{3}\left[\cos 3t + \sin 3t\right]$$

$$x'(t) = -\frac{1}{2}e^{-t/2}\left[C_1\cos\left(\frac{\sqrt{47}}{2}t\right) + C_2\sin\left(\frac{\sqrt{47}}{2}t\right)\right] +$$

$$e^{-t/2}\left[-C_1\frac{\sqrt{47}}{2}\sin\left(\frac{\sqrt{47}}{2}t\right) + \frac{\sqrt{47}}{2}C_2\cos\left(\frac{\sqrt{47}}{2}t\right)\right]$$

$$+\frac{10}{3}\left[-3\sin 3t + 3\cos 3t\right]$$

$$x(0) = 2 \implies 1\left[C_1 + 0\right] + \frac{10}{3}\left[1+0\right] = 2$$

$$C_1 = 2 - \frac{10}{3} = \frac{6-10}{3} = -\frac{4}{3} ;\ \left(C_1 = -\frac{4}{3}\right)$$

$$x'(0) = 0$$

$$-\frac{1}{2}\left[-\frac{4}{3} + 0\right] + 1\left[0 + \frac{\sqrt{47}}{2}C_2\right] + \frac{10}{3}\left[0+3\right] = 0$$

$$\frac{2}{3} + \frac{\sqrt{47}}{2}C_2 + 10 = 0$$

$$\frac{\sqrt{47}}{2}C_2 = -\frac{32}{3} \implies C_2 = -\frac{64}{3\sqrt{47}} = \frac{-64\sqrt{47}}{3\times 47}$$

$$\therefore X_{complete}(t) = e^{-t/2}\left[-\frac{4}{3}\cos\left(\frac{\sqrt{47}}{2}t\right) - \frac{64\sqrt{47}}{141}\sin\left(\frac{\sqrt{47}}{2}t\right)\right] + \frac{10}{3}\left[\cos 3t + \sin 3t\right].$$

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#3) Mass = 1 slug , $k = \frac{32}{2} = 16 \frac{lbf}{ft}$, $f(t) = 8 \sin 4t$

Damping force $= 8 \frac{dx}{dt}$

(1) $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 16x = 8\sin 4t$; $x(0) = 0$, $x'(0) = 0$

$r^2 + 8r + 16 = 0$

$(r+4)^2 = 0 \implies r = -4$

$\therefore X_h(t) = C_1 e^{-4t} + C_2 t e^{-4t}$

$X_p = A\sin 4t + B\cos 4t$

$X_p' = 4A\cos 4t - 4B\sin 4t$

$X_p'' = -16A\sin 4t - 16B\cos 4t$

Substitute back into (1)

$-16A\sin 4t - 16B\cos 4t + 8(4A\cos 4t - 4B\sin 4t) + 16(A\sin 4t + B\cos 4t) = 8\sin 4t$

$(-16A - 32B + 16A)\sin 4t + (-16B + 32A + 16B)\cos 4t = 8\sin 4t$

$-32B = 8 \implies B = -1/4$

$32A = 0 \implies A = 0$

$\therefore X_{complete}(t) = C_1 e^{-4t} + C_2 t e^{-4t} - \frac{1}{4}\cos 4t$

$x'(t) = -4C_1 e^{-4t} + C_2 e^{-4t} - 4C_2 t e^{-4t} + \sin 4t$

$x(0) = 0 \implies C_1 - \frac{1}{4} = 0 \implies \boxed{C_1 = 1/4}$

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#3) $x'(0)=0$, Starting From Rest

$\Rightarrow -4c_1 + c_2 = 0 \Rightarrow c_2 = 4c_1 = 1$; $\boxed{c_2 = 1}$

$\therefore X_{complete}(t) = \frac{1}{4} e^{-4t} + t e^{-4t} - \frac{1}{4} \cos 4t$