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#44) $4y'' - 4y' - 3y = 0 \quad ; \; y(0) = 1, \; y'(0) = 5$

$4r^2 - 4r - 3 = 0$

$(2r-3)(2r+1) = 0 \implies r = 3/2, -1/2$

$y(x) = c_1 e^{\frac{3}{2}x} + c_2 e^{-\frac{1}{2}x} \quad ; \quad y'(x) = \frac{3}{2} c_1 e^{3/2 x} - \frac{1}{2} c_2 e^{-1/2 x}$

$y(0) = 1 \implies c_1 + c_2 = 1$

$y'(0) = 5 \qquad \frac{3}{2} c_1 - \frac{1}{2} c_2 = 5$

$$c_1 = \frac{\begin{vmatrix} 1 & 1 \\ 5 & -1/2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 3/2 & -1/2 \end{vmatrix}} = \frac{-1/2 - 5}{-1/2 - 3/2} = \frac{-1-10}{-1-3} = \frac{11}{4}$$

$$c_2 = \frac{\begin{vmatrix} 1 & 1 \\ 3/2 & 5 \end{vmatrix}}{-2} = \frac{5 - 3/2}{-2} = \frac{10-3}{-4} = -\frac{7}{4}$$

$\therefore y(x) = \frac{11}{4} e^{3/2 x} - \frac{7}{4} e^{-1/2 x}$

$$\boxed{y(x) = \frac{11}{4} e^{\frac{3x}{2}} - \frac{7}{4} e^{-x/2}}$$

#54) $y'' + 4y = 0 \quad , \; y(0) = 0, \; y(\pi) = 0$

$r^2 + 4 = 0 \implies r = \pm 2i$

$y(x) = c_1 \cos 2x + c_2 \sin 2x \quad ; \quad y'(x) = -2c_1 \sin 2x + 2c_2 \cos 2x$

$y(0) = 0 \implies \boxed{c_1 = 0}$

$y(\pi) = 0 \implies c_2 \sin 2\pi = 0 \quad ; \text{ But } \sin 2\pi = 0$

$\implies c_2$ is a free variable constant.

$$\boxed{\therefore y(x) = c_2 \sin 2x}$$

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#30)

$$2y''+3y'-2y = 14x^2-4x-11, \quad y(0)=0, \; y'(0)=0$$

The associated homogeneous equation is:

$$2y''+3y'-2y=0$$
$$2r^2+3r-2=0$$
$$(2r-1)(r+2)=0 \implies r = 1/2, -2$$

$$\therefore y_H = C_1e^{x/2}+C_2e^{-2x}$$

$$y_{particular} = ax^2+bx+c$$
$$y_p' = 2ax+b, \quad y_p'' = 2a$$

$$2\cdot 2a + 3(2ax+b) - 2(ax^2+bx+c) = 14x^2-4x-11$$

$$4a + 6ax + 3b - 2ax^2 - 2bx - 2c = 14x^2-4x-11$$

$$\implies -2ax^2 + (6a-2b)x + 4a+3b-2c = 14x^2-4x-11$$

$$\implies -2a = 14 \implies \boxed{a=-7}$$

$$6a-2b = -4 \implies b = \frac{-4-6a}{-2}$$

$$b = \frac{4-42}{2} = -19$$

$$4a+3b-2c = -11$$

$$c = \frac{-11-4a-3b}{-2} = \frac{11-28-57}{2} = \frac{-74}{2} = -37$$

$$\therefore y_p = -7x^2-19x-37$$

$$y_{total} = y_H+y_p = C_1e^{x/2}+C_2e^{-2x}-7x^2-19x-37$$

$$y'(x) = \frac{C_1}{2}e^{x/2} - 2C_2e^{-2x} - 14x - 19$$

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Now we adapt to the initial conditions.

$y(0)=0 \implies C_1+C_2-37=0$

$y'(0)=0 \implies \frac{C_1}{2}-2C_2-19=0$

$$\implies \begin{aligned} C_1+C_2 &= 37 \\ \frac{C_1}{2}-2C_2 &= 19 \end{aligned}$$

$$C_1 = \frac{\begin{vmatrix} 37 & 1 \\ 19 & -2 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 1/2 & -2 \end{vmatrix}} = \frac{-74-19}{-2-1/2} = 93 \times \frac{2}{5} = \frac{186}{5}$$

$$\boxed{C_1 = \frac{186}{5}}$$

$$C_2 = \frac{\begin{vmatrix} 1 & 37 \\ 1/2 & 19 \end{vmatrix}}{-5/2} = \frac{19-37/2}{-5/2} = \frac{38-37}{-5} = -1/5$$

$$\boxed{C_2 = -1/5}$$

$$\boxed{\therefore\ y_{total}(x) = \frac{186}{5}e^{x/2} - \frac{1}{5}e^{-2x} - 7x^2 - 19x - 37}$$

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Section 4.7

#6) $y'' + y = \sec^2 x$

The associated homogeneous equation is:

$y'' + y = 0$

$\Rightarrow r^2 + 1 = 0 \Rightarrow r = \pm i$

$y_H = c_1 \cos x + c_2 \sin x \quad ; y_1 = \cos x, \ y_2 = \sin x$

$y_p = u_1(x) y_1 + u_2(x) y_2$ where

$$\left.\begin{array}{l} u_1' y_1 + u_2' y_2 = 0 \\ u_1' y_1' + u_2' y_2' = \sec^2 x \end{array}\right\} \Rightarrow \begin{array}{l} u_1' \cos x + u_2' \sin x = 0 \\ -u_1' \sin x + u_2' \cos x = \sec^2 x \end{array}$$

$$\Rightarrow u_1' = \frac{\begin{vmatrix} 0 & \sin x \\ \sec^2 x & \cos x \end{vmatrix}}{\begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}} = \frac{-\sin x \sec^2 x}{\cos^2 x + \sin^2 x}$$

$$u_1' = -\sec x \tan x$$

$$\Rightarrow u_1(x) = -\int \sec x \tan x \, dx = -\sec x$$

$$u_2'(x) = \frac{\begin{vmatrix} \cos x & 0 \\ -\sin x & \sec^2 x \end{vmatrix}}{1} = \cos x \sec^2 x = \sec x$$

$u_2'(x) = \sec x$

$\Rightarrow u_2(x) = \int \sec x \, dx = \ln|\sec x + \tan x|$

$\therefore y_p(x) = -\sec x \cos x + \ln|\sec x + \tan x| \sin x = -1 + \ln|\sec x + \tan x| \sin x$

$y_{total} = c_1 \cos x + c_2 \sin x - 1 + (\ln|\sec x + \tan x|) \sin x$