Dr. ZABDAWI

Section 3.3

#1)

$$\frac{dP}{dt} = kP$$

$$\int \frac{dP}{P} = \int k\,dt$$

$$\ln|P| = kt + \ln C$$

$$\Rightarrow \ln\left|\frac{P}{C}\right| = kt$$

$$\Rightarrow P(t) = Ce^{kt}$$

@ $t=0$, $P(0) = P_0 \Rightarrow C = P_0$

$$P(t) = P_0 e^{kt}$$

@ $t=5$, $P(5) = 2P_0 = P_0 e^{5k}$

$$\Rightarrow 2 = e^{5k}$$

$$\ln 2 = 5k \Rightarrow k = \frac{\ln 2}{5}$$

$$P(t) = P_0 e^{\frac{\ln 2}{5}t}$$

a) $t = ?$ for $P(t) = 3P_0$

$$\rightarrow 3P_0 = P_0 e^{\frac{\ln 2}{5}t}$$

$$3 = e^{\left(\frac{\ln 2}{5}t\right)}$$

$$\ln 3 = \frac{\ln 2}{5}t \Rightarrow t = \frac{5\ln 3}{\ln 2} = 7.92 \text{ Years}$$

b) $t = ?$ for $\frac{P(t)}{P_0} = 4$

$$\Rightarrow 4 = e^{\frac{\ln 2}{5}t}$$

$$\ln 4 = \frac{\ln 2}{5}t \Rightarrow t = \frac{5\ln 4}{\ln 2} = \frac{5\times 2\ln 2}{\ln 2} = 10 \text{ Years.}$$

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#2) From problem #(1) we have:

$$P(t) = P_0 e^{\frac{\ln 2}{5} t}$$

@ $t=3$, $P(3) = 10,000$

$$\Rightarrow P_0 e^{\left(\frac{\ln 2}{5} \times 3\right)} = 10,000$$

$$P_0 = \frac{10,000}{e^{\left(\frac{3}{5}\ln 2\right)}} = 6597.5$$

$\rightarrow$ @ $t=10$ $P(10) = 4P_0 = 4 \times 6597.5 = \boxed{26390 \text{ people}}$

#11)

$$A(t) = A_0 e^{-\frac{\ln 2}{5600} t}$$

85.5% of C-14 had decayed $\Rightarrow \frac{A(t)}{A_0} = (1 - .855) = .145$

$$\Rightarrow .145 = e^{-\frac{\ln 2}{5600} t}$$

$$\ln .145 = -\frac{\ln 2}{5600} t$$

$$\Rightarrow t = -\frac{(\ln .145)(5600)}{\ln 2} = 15600.901$$

$$\simeq 15,600 \text{ Years}$$

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Section 4.1

#6) $y = c_1 + c_2 x^2$ is a solution to $xy'' - y' = 0$ on $x \in (-\infty, +\infty)$

$y(0) = 0 \implies \boxed{c_1 = 0}$

$y'(0) = 1 \implies 2c_2 x\big|_{x=0} = 0 \neq 1 \longrightarrow c_2$ cannot be determined.

This doesn't violate the Existence and Uniqueness Theorem (4.1) because $a_n(x) = x = 0$ @ $x = 0$.

#22) $y_1 = x$, $y_2 = x\ln x$, $y_3 = x^2 \ln x$; $x \in (0, \infty)$

$y_1' = 1$, $y_1'' = 0$

$y_2 = x\ln x$, $y_2' = \ln x + 1$, $y_2'' = 1/x$

$y_3 = x^2 \ln x$, $y_3' = 2x\ln x + x$, $y_3'' = 2\ln x + 2 + 1 = 2\ln x + 3$

$$W(y_1, y_2, y_3) = \begin{vmatrix} x & x\ln x & x^2 \ln x \\ 1 & \ln x + 1 & 2x\ln x + x \\ 0 & 1/x & 2\ln x + 3 \end{vmatrix}$$

$= x\left[(\ln x + 1)(2\ln x + 3) - \frac{1}{x}(2x\ln x + x)\right] - x\ln x(2\ln x + 3) + x^2 \ln x \cdot \frac{1}{x}$

$= x\left[2\ln^2 x + 3\ln x + 2\ln x + 3 - 2\ln x - 1\right] - 2x\ln^2 x - 3x\ln x + x\ln x$

$= 2x\ln^2 x + 3x\ln x + 2x - 2x\ln^2 x - 3x\ln x + x\ln x$

$= 2x + x\ln x = x(2 + \ln x) \neq 0$ for $x \in (0, +\infty)$

$\therefore$ x, $x\ln x$, $x^2\ln x$ are linearly independent