Section 3.2

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#6)
$$\frac{dx}{dt} = kx$$
$$\int \frac{dx}{x} = \int k\,dt$$
$$\ln|x| = kt + \ln c$$
$$\rightarrow X(t) = ce^{kt}$$
where $X(t)$ represents the amount remaining at time t.

@ $t=0$, $X(0) = 100 \rightarrow C = 100$
$$X(t) = 100e^{kt}$$

@ $t = 6$ Hours $X(t) = 97$, 3% decrease
$$\rightarrow .97 = e^{6k} \rightarrow k = \frac{\ln .97}{6} = -5.077 \times 10^{-3}$$
$$\therefore X(t) = 100e^{-5.077 \times 10^{-3} t}$$

@ $t = 24$, $X(24) = 100e^{(-5.077 \times 10^{-3} \times 24)}$
$$= \boxed{88.53 \text{ mg.}}$$

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#12) Newton Law of Cooling / Heating

$$\frac{dT}{dt} = k(T - T_\infty), \quad T_\infty = \text{Ambient Temperature}$$

$$T_\infty = 5$$

$$\int \frac{dT}{(T - T_\infty)} = \int k\,dt$$

$$\ln|T - T_\infty| = kt + \ln c$$

$$(T - T_\infty) = c e^{kt}$$

@ $t = 0$, $T = T_0$ = Initial Temperature of the room

$$\Rightarrow (T_0 - T_\infty) = C$$

$$\therefore T(t) = (T_0 - T_\infty)e^{kt} + T_\infty$$

$$T(t) = (T_0 - 5)e^{kt} + 5$$

Our job to find T_0.
We have some experimental data

@ $t = 1$ minute, $T(1) = 55°F$
@ $t = 5$ minute, $T(5) = 30°F$

$$\Rightarrow 55 - 5 = (T_0 - 5)e^{k} \quad \text{——— (1)}$$

$$= (T_0 - 5)e^{5k} \quad \text{——— (2)}$$

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$$50 = (T_0 - 5)\, e^{k} \quad —(1)$$
$$25 = (T_0 - 5)\, e^{5k} \quad —(2)$$
$$\frac{(1)}{(2)} \Rightarrow 2 = e^{-4k}$$

$$\ln 2 = -4k \Rightarrow k = -\frac{\ln 2}{4} = -1{,}733 \times 10^{-1}$$

Using (1) $\Rightarrow$ $50 = (T_0 - 5)\, e^{-1{,}733 \times 10^{-1}}$

$$T_0 = 50\, e^{1{,}733 \times 10^{-1}} + 5$$

$$\boxed{T_0 = 64{,}46^{\circ} \simeq 65^{\circ} F.}$$