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#4) $\left(2+e^{-x/y}\right)dx + 2\left(1-\frac{x}{y}\right)dy = 0$

let $u = \frac{x}{y} \Rightarrow x = yu$
$dx = u\,dy + y\,du$

$\Rightarrow \left(2+e^{-u}\right)\left(u\,dy + y\,du\right) + 2(1-u)\,dy = 0$

$\left(2+e^{-u}\right)u\,dy + \left(2+e^{-u}\right)y\,du + 2(1-u)\,dy = 0$

$\left(2+ue^{-u}\right)dy + \left(2+e^{-u}\right)y\,du = 0$

$\frac{dy}{y} = -\frac{\left(2+e^{-u}\right)du}{\left(2+ue^{-u}\right)} \times \left(\frac{e^{u}}{e^{u}}\right) \; ; e^{u} \neq 0$

$\frac{dy}{y} = -\frac{\left(2e^{u}+1\right)du}{\left(2e^{u}+u\right)}$

let $t = 2e^{u}+u \Rightarrow dt = \left(2e^{u}+1\right)du$

$\Rightarrow \int \frac{dy}{y} = -\int \frac{dt}{t}$

$\ln|y| = -\ln|t| + \ln c$

$\Rightarrow \ln(ty) = \ln c$

$\Rightarrow ty = c$

$\left(2e^{u}+u\right)y = c$

$\Rightarrow \boxed{y = \frac{c}{\left(2e^{x/y}+\frac{x}{y}\right)}}$

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#2) $y' = y + x(y+1)^2 + 1$

$\frac{dy}{dx} = (y+1) + x(y+1)^2$

let $u = y+1 \Rightarrow \frac{du}{dx} = \frac{dy}{dx}$

$\frac{du}{dx} = u + x u^2$

(2) $\frac{du}{dx} - u = x u^2$; This is Bernoulli's Eq. with $n=2$

let $w = u^{1-2} = \frac{1}{u} \Rightarrow \frac{dw}{dx} = -\frac{1}{u^2}\frac{du}{dx}$

Eq(2) $\Rightarrow$ $\frac{1}{u^2}\frac{du}{dx} - \frac{1}{u} = x$

$-\frac{dw}{dx} - w = x$

$\frac{dw}{dx} + w = -x$

$M_q = e^{\int 1\,dx} = e^x$

$e^x \frac{dw}{dx} + e^x w = -x e^x$

$(e^x w)' = -x e^x$

$e^x w = -\int x e^x dx$

let $u = x \Rightarrow du = dx$

$dv = e^x dx \Rightarrow v = e^x$

$\rightarrow e^x w = -[x e^x - \int e^x dx] + C$

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#8)

$$e^x w = -[xe^x - e^x] + C$$

$$w = -(x-1) + Ce^{-x} \qquad ; \; w = \frac{1}{u} = \frac{1}{y+1}$$

$$\frac{1}{y+1} = -(x-1) + Ce^{-x}$$

$$y+1 = \frac{1}{Ce^{-x}+1-x}$$

$$y = \frac{1}{Ce^{-x}+1-x} - 1$$

$$\boxed{y = (Ce^{-x}+1-x)^{-1} - 1}$$

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#5) $\frac{dy}{dx} - \frac{4}{x}y = 2x^5 e^{y/x^4}$ —— (1)

let $u = \frac{y}{x^4}$

$\Rightarrow \frac{du}{dx} = \frac{x^4 \cdot dy/dx - 4x^3 \cdot y}{x^8} = \frac{1}{x^4}\frac{dy}{dx} - \frac{4}{x^5}y$

$\frac{du}{dx} = \frac{1}{x^4}\frac{dy}{dx} - \frac{4}{x^5}y$ —— (2)

Eq.(1) $\Rightarrow \frac{1}{x^4}\frac{dy}{dx} - \frac{4}{x^5}y = 2x e^{y/x^4}$; $x \neq 0$

Eq.(2) $\Rightarrow \frac{du}{dx} = 2x e^{u}$

$\int \frac{du}{e^u} = \int 2x\,dx$

$-e^{-u} = x^2 + C$

$\Rightarrow -e^{-y/x^4} = x^2 + C.$