Dr. Zaid Alwan

Section 2.6

#4) $x\frac{dy}{dx} - (1+x)y = xy^2$ (1) ; $x \neq 0$

$\frac{dy}{dx} - \left(\frac{1+x}{x}\right)y = y^2$

Let $w = y^{-1}$ $\Rightarrow \frac{dw}{dx} = -y^{-2}\frac{dy}{dx}$

$\Rightarrow \frac{1}{y^2}\frac{dy}{dx} = -\frac{dw}{dx}$

Eq. (1) $\Rightarrow$ $\frac{1}{y^2}\frac{dy}{dx} - \frac{(1+x)}{xy} = 1$

$-\frac{dw}{dx} - \frac{(1+x)}{x}w = 1$

$\frac{dw}{dx} + \left(\frac{1+x}{x}\right)w = -1$

$\mu(x) = e^{\int \left(\frac{1+x}{x}\right)dx} = e^{(\ln x + x)} = x \cdot e^x$

$\Rightarrow xe^x\frac{dw}{dx} + (1+x)e^x w = -xe^x$

$(xe^x w)' = -xe^x$

$xe^x w = -\int xe^x dx$ —(2)

Let $u = x \Rightarrow du = dx$

$dv = e^x dx \Rightarrow v = e^x$

(2) $\Rightarrow$ $xe^x w = -\left[xe^x - \int e^x dx\right] + C$

$xe^x w = -xe^x + e^x + C$

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#4) $xe^x w = -xe^x + e^x + C$

$w = -1 + \frac{1}{x} + \frac{C}{xe^x}$; $w = \frac{1}{y}$

$\frac{1}{y} = -1 + \frac{1}{x} + \frac{C}{xe^x}$

$= \frac{-xe^x + e^x + C}{xe^x}$

$$\boxed{y = \frac{xe^x}{-xe^x + e^x + C}}$$

#10) $y^{1/2}\frac{dy}{dx} + y^{3/2} = 1$ —— (1) ; $y(0) = 4$

$\frac{dy}{dx} + y = y^{-1/2}$ —— (2)

Let $w = y^{3/2}$ $\rightarrow \frac{dw}{dx} = \frac{3}{2} y^{1/2}\frac{dy}{dx}$

$\rightarrow y^{1/2}\frac{dy}{dx} = \frac{2}{3}\frac{dw}{dx}$

(1) $\Rightarrow$ $\frac{2}{3}\frac{dw}{dx} + w = 1$

$\frac{dw}{dx} + \frac{3}{2}w = \frac{3}{2}$

$\mu(x) = e^{\int 3/2\, dx} = e^{\frac{3}{2}x}$

$e^{3/2x}\frac{dw}{dx} + \frac{3}{2}e^{3/2x}w = \frac{3}{2}e^{3/2x}$

$(e^{3/2x}w)' = \frac{3}{2}e^{\frac{3}{2}x}$

$e^{\frac{3}{2}x}w = \frac{3}{2}\int e^{3/2x}dx$

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#8) $e^{3/2x} W = \frac{3}{2} e^{3/2x} \cdot \frac{2}{3} + C$

$W = 1 + Ce^{-\frac{3}{2}x}$

$y^{3/2} = 1 + Ce^{-\frac{3}{2}x}$

Given I.C. $y(0) = 4$

$\Rightarrow 4^{3/2} = 1 + C \rightarrow C = 4^{3/2} - 1 = 8 - 1 = 7$

$\therefore y^{3/2} = 1 + 7e^{-3/2x}$