Dr. ZABDAWI

Section 2.4

#26) $(e^x+y)dx+(2+x+ye^y)dy=0 \quad , y(0)=1$

$M=e^x+y, \quad N=2+x+ye^y$

$\frac{\partial M}{\partial y}=1=\frac{\partial N}{\partial x}=1 \longrightarrow$ the eq. is exact.

$\frac{\partial f}{\partial x}=e^x+y \longrightarrow f(x,y)=e^x+yx+g(y)$

$\frac{\partial f}{\partial y}=x+\frac{dg}{dy}=2+x+ye^y$

$\Longrightarrow \frac{dg}{dy}=2+ye^y$

$g(y)=\int(2+ye^y)dy=2y+\int ye^y dy$

Integrate By Parts: let $u=y \longrightarrow du=dy$

$dV=e^y dy \longrightarrow V=e^y$

$g(y)=2y+ye^y-\int e^y dy$

$=2y+ye^y-e^y+c_1$

$\therefore f(x,y)=e^x+yx+2y+ye^y-e^y+c_1$

$f(x,y)=c_2$

$\longrightarrow e^x+yx+2y+ye^y-e^y=c \quad$; where $c=c_2-c_1$

$y(0)=1 \longrightarrow 1+0+2+1e^1-e^1=c \longrightarrow \boxed{c=3}$

$\therefore \boxed{e^x+yx+2y+ye^y-e^y=3}$

Section 2.4 Dr. ZABBDAWI

#34) $(6xy^3 + \cos y)\,dx + (kx^2y^2 - x\sin y)\,dy = 0$

$M = 6xy^3 + \cos y \quad , N = kx^2y^2 - x\sin y$

$\frac{\partial M}{\partial y} = 18xy^2 - \sin y \quad , \quad \frac{\partial N}{\partial x} = 2kxy^2 - \sin y$

for the equation to be exact . $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

$\Rightarrow \quad 2k = 18 \Rightarrow \boxed{k = 9}$

$\frac{\partial f}{\partial x} = M = 6xy^3 + \cos y$

$\Rightarrow f(x,y) = \frac{6x^2y^3}{2} + x\cos y + g(y)$

$= 3x^2y^3 + x\cos y + g(y)$

$\frac{\partial f}{\partial y} = 9x^2y^2 - x\sin y + \frac{dg}{dy} = N = 9x^2y^2 - x\sin y$

$\Rightarrow \frac{dg}{dy} = 0 \Rightarrow g(y) = c_1$

$f(x,y) = c_2$

$\Rightarrow \quad 3x^2y^3 + x\cos y + c_1 = c_2$

$\Rightarrow \quad \boxed{3x^2y^3 + x\cos y = c}$; where $c = c_2 - c_1$