Dr. ZABIDAWI

Section 2.3

#17) $\frac{dy}{dx} = \frac{y-x}{y+x}$

Eq. is homogeneous of degree (1).

let $y = u(x) \cdot x$

$y' = u'x + u$

$$u'x + u = \frac{ux - x}{ux + x} = \frac{x(u-1)}{x(u+1)} \quad ; x \neq 0$$

$$u'x = \frac{u-1}{u+1} - u = \frac{u-1-u(u+1)}{u+1} = -\frac{(1+u^2)}{(u+1)}$$

$$\frac{(u+1)\,du}{(1+u^2)} = -\frac{dx}{x}$$

$$\int \left(\frac{u}{1+u^2} + \frac{1}{1+u^2}\right) du = -\int \frac{dx}{x}$$

$$\tfrac{1}{2} \ln|1+u^2| + \operatorname{Arctan} u = -\ln|x| + \ln c = \ln\left|\frac{c}{x}\right|$$

$$\Rightarrow \tfrac{1}{2} \ln\left|1+\left(\tfrac{y}{x}\right)^2\right| + \operatorname{Arctan}\left(\tfrac{y}{x}\right) = \ln\left|\frac{c}{x}\right| \quad ; \text{This answer is good.}$$

$$\ln\left|\frac{x^2+y^2}{x^2}\right| + 2\tan^{-1}\left(\tfrac{y}{x}\right) = 2\ln c - 2\ln x$$

$$\ln(x^2+y^2) - \ln x^2 + 2\tan^{-1}\left(\tfrac{y}{x}\right) = 2\ln c - 2\ln x$$

$$\Rightarrow \ln(x^2+y^2) + 2\tan^{-1}\left(\tfrac{y}{x}\right) = 2\ln c - 2\ln x + 2\ln x \quad ; \text{let } C = 2\ln c$$

$$\therefore \boxed{\ln(x^2+y^2) + 2\tan^{-1}\left(\tfrac{y}{x}\right) = C}$$

Dr. ZABDAWI

Section 2.3

#39) $(x+\sqrt{xy})\frac{dy}{dx} + x - y = x^{-1/2}y^{3/2}$, $y(1) = 1$

$$\frac{dy}{dx} = \frac{y - x + x^{-1/2}y^{3/2}}{(x+\sqrt{xy})}$$ Eq. is Homogeneous of degree (1).

let $y = u(x) \cdot x$

$$\frac{dy}{dx} = u'x + u$$

$$u'x + u = \frac{ux - x + x^{-1/2}(ux)^{3/2}}{x + \sqrt{x \cdot ux}}$$

$$u'x + u = \frac{ux - x + x \cdot u^{3/2}}{x + x\sqrt{u}} = \frac{x(u - 1 + u^{3/2})}{x(1+\sqrt{u})} ;\ x \neq 0$$

$$u'x + u = \frac{u - 1 + u^{3/2}}{1+\sqrt{u}}$$; $x > 0$ because $\sqrt{x^2} = |x| = x$ if $x > 0$

$$u'x = \frac{u - 1 + u^{3/2} - u(1+\sqrt{u})}{1+\sqrt{u}}$$

$$u'x = \frac{u - 1 + u^{3/2} - u + u^{3/2}}{1+\sqrt{u}} = \frac{-1}{1+u^{1/2}}$$

$$(1+u^{1/2})\,du = -\frac{dx}{x}$$

$$u + u^{3/2} \cdot \frac{2}{3} = -\ln|x| + \ln c = \ln\left|\frac{c}{x}\right|$$

$$\frac{y}{x} + \frac{2}{3}\left(\frac{y}{x}\right)^{3/2} = \ln\left|\frac{c}{x}\right| \quad ;\ y(1) = 1$$

$$1 + \frac{2}{3} = \ln|c| \implies \boxed{c = e^{5/3}}$$

Dr. BABDAUL Continue #39 Section 2.3

#39) $\frac{y}{x} + \frac{2}{3}\left(\frac{y}{x}\right)^{3/2} = \frac{5}{3} - \ln|x|$; This answer is good.

To get the answer in the back of the book multiply each side by $3x^{3/2}$

$\Rightarrow 3x^{3/2}\frac{y}{x} + 2y^{3/2} = 5x^{3/2} - 3x^{3/2}\ln|x|$

$\Rightarrow 3x^{1/2}y + 2y^{3/2} + 3x^{3/2}\ln|x| = 5x^{3/2}$

H.W. For Section 2.3 : Homogeneous eq. 1/11/05

#32) $(x^2+2y^2)dx = xy\,dy$, $y(-1)=1$

$(x^2+2y^2)dx - xy\,dy = 0$

$$\frac{dy}{dx} = \frac{(x^2+2y^2)}{xy} \quad ; \text{ Homogeneous.}$$

let $y = u(x)\,x$

$$\frac{dy}{dx} = u'x + u$$

$$u'x + u = \frac{x^2+2u^2x^2}{x^2u} \quad , x \neq 0$$

$$u'x + u = \frac{1+2u^2}{u}$$

$$u'x = \frac{1+2u^2}{u} - u = \frac{1+u^2}{u}$$

$$\frac{u\,du}{1+u^2} = \frac{dx}{x}$$

let $t = 1+u^2 \Longrightarrow dt = 2u\,du$

$$\frac{1}{2}\frac{dt}{t} = \frac{dx}{x}$$

$$\frac{1}{2}\ln|t| = \ln|x| + \ln c_1$$

$$\ln|t| = 2\ln|x| + \ln c \quad , \quad \ln c = 2\ln c_1$$

$$\ln|t| - \ln|x^2| = \ln c$$

$$\frac{t}{x^2} = c$$

$$\Longrightarrow \frac{1+\left(\frac{y}{x}\right)^2}{x^2} = c$$

$$\frac{x^2+y^2}{x^4} = c \Longrightarrow x^2+y^2 = cx^4$$

$$\frac{x^2+y^2}{x^4} = c$$

$$x^2+y^2 = Cx^4 \qquad y(-1)=1 \longrightarrow (-1,1)$$

$$1+1 = C(-1)^4 \longrightarrow \boxed{C=2}$$

$$x^2+y^2 = 2x^4$$

#35) $\left(x+ye^{y/x}\right)dx - xe^{y/x}dy = 0 \quad , \; y(1)=0$

$$\frac{dy}{dx} = \frac{x+ye^{y/x}}{xe^{y/x}} \qquad \text{Homogeneous.}$$

Let $y = u(x)\,x \Longrightarrow \frac{dy}{dx} = u'x+u$

$$u'x+u = \frac{x+xue^{u}}{xe^{u}} \; , \quad x \neq 0$$

$$u'x+u = \frac{1+ue^{u}}{e^{u}}$$

$$u'x = \frac{1+ue^{u}}{e^{u}} - u = \frac{1}{e^{u}}$$

$$e^{u}\,du = \frac{dx}{x}$$

$$e^{u} = \ln|x| + c$$

$$e^{y/x} = \ln|x|+c \; , \quad y(1)=0 \longrightarrow (1,0)$$

$$1 = \ln|1| + c \longrightarrow \boxed{c=1}$$

$\therefore \; e^{y/x} = \ln|x| + 1 \qquad$ Implicit Solution.