Section 2.2 Separable Variables

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#12) $\dfrac{dx}{dy} = \dfrac{1+2y^2}{y\sin x}$

$\Longrightarrow dy\,\dfrac{(1+2y^2)}{y} = \sin x\,dx$

$\displaystyle\int (y^{-1}+2y)\,dy = \int \sin x\,dx$

$\ln|y| + y^2 = -\cos x + C$

#18) $x^2y^2\,dy = (y+1)\,dx$

$\dfrac{y^2}{(y+1)}\,dy = \dfrac{dx}{x^2}$,

$$\begin{array}{r|l} & y-1 \\ \hline y+1 & y^2 \\ & y^2+y \\ \hline & -y \\ & -y-1 \\ \hline & +1 \end{array}$$

So $\dfrac{y^2}{y+1} = (y-1) + \dfrac{1}{(y+1)}$

$\displaystyle\int \left[(y-1) + \frac{1}{(y+1)}\right] dy = \int x^{-2}\,dx$

$\dfrac{y^2}{2} - y + \ln|y+1| = \dfrac{1}{x} + C$

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#34) $\dfrac{dy}{dx} = \dfrac{xy+2y-x-2}{xy-3y+x-3} = \dfrac{y(x+2)-(x+2)}{y(x-3)+(x-3)}$

$$\frac{dy}{dx} = \frac{(x+2)[y-1]}{(x-3)[y+1]}$$

$$\left(\frac{y+1}{y-1}\right)dy = \left(\frac{x+2}{x-3}\right)dx$$

$$\begin{array}{r|l} & 1 \\ \hline y-1 & y+1 \\ & y-1 \\ \hline & 2 \end{array} \quad , \quad \begin{array}{r|l} & 1 \\ \hline x-3 & x+2 \\ & x-3 \\ \hline & 5 \end{array}$$

$$\rightarrow \int\left(1+\frac{2}{y-1}\right)dy = \int\left[1+\left(\frac{5}{x-3}\right)\right]dx$$

$$y + 2\ln|y-1| = x + 5\ln|x-3| + C$$

#50) $x\dfrac{dy}{dx} = y^2 - y$

$$\int \frac{1}{(y^2-y)}\,dy = \int \frac{dx}{x}$$

Here we have to integrate $\int \frac{1}{(y^2-y)}dy$ By Partial Fractions.

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$$\frac{1}{y^2-y} = \frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1}$$

$$A = \left.\frac{1}{(y-1)}\right|_{y=0} = -1$$

$$B = \left.\frac{1}{y}\right|_{y=1} = 1$$

$$\therefore \int \frac{1}{(y^2-y)}\,dy = \int \frac{dx}{x}$$

$$\int\left(\frac{-1}{y} + \frac{1}{(y-1)}\right)dy = \int \frac{dx}{x}$$

$$-\ln|y| + \ln|y-1| = \ln|x| + \ln c$$

$$\frac{y-1}{y} = cx$$

a) $(0,1) \Rightarrow \frac{1-1}{1} = c\cdot 0$ $\quad \underline{(0=0)}$

$$y-1 = cyx$$

$$y(1-cx) = 1 \rightarrow y = \frac{1}{1-cx}$$

$y(0)=1 \rightarrow y=1$ is a solution.

Also notice from the beginning that

$$x\frac{dy}{dx} = y^2 - y$$

has $y=0$ as a trivial solution

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(Continue #50)

b) $(0,0) \Rightarrow y(0)=0 \Rightarrow y=0$ as as we mentioned the trivial solution.

c) $(1/2, 1/2) \rightarrow y(1/2) = 1/2$

$$y = \frac{1}{1-cx}$$

$$\rightarrow 1/2 = \frac{1}{1-c/2}$$

$$\Rightarrow 1-\frac{c}{2} = \frac{1}{1/2} = 2$$

$$\frac{c}{2} = -1 \Rightarrow c = -2$$

$$\therefore \boxed{y = \frac{1}{1+2x}}$$