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Ex 5 $\frac{dy}{dx} = x^2+y^2 = f(x,y)$

$f(x,y) = x^2+y^2$ Cont

$\frac{\partial f}{\partial y} = 2y$ Cont on $(-\infty,+\infty)$

∴ through any given (x_0,y_0) there passes or exist sol. to the differential eq.

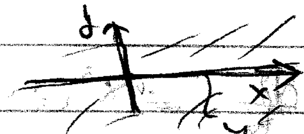
Sol. to H.W. Section 2.1

#1) $\frac{dy}{dx} = y^{2/3} = f(x,y)$

$f(x,y) = \sqrt[3]{y^2}$ Cont. on $(-\infty,+\infty)$

$\frac{\partial f}{\partial y} = \frac{2}{3}y^{-1/3} = \frac{2}{3\sqrt[3]{y}}$ Cont. for $y \neq 0$.

$y \in (-\infty,0)\cup(0,+\infty)$



$y \neq 0$

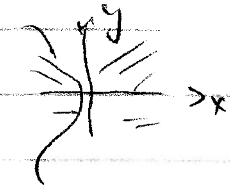
#3) $x\frac{dy}{dx} = y \to \frac{dy}{dx} = y/x = f(x,y)$

$f(x,y) = y/x$ Cont. $y \in (-\infty,+\infty)$

$x \in (-\infty,0)\cup(0,+\infty)$

$\frac{\partial f}{\partial y} = 1/x$ Cont $x \neq 0$

E.U. a Half plane $x>0$ or $x<0$



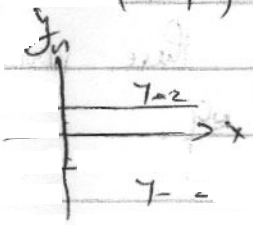
$x \neq 0$

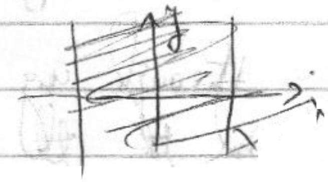
#5) $(4-y^2)\, y' = x^2$

$y' = \frac{dy}{dx} = \frac{x^2}{(4-y^2)} = f(x,y)$

$f(x,y)$ is cont for $y \neq \pm 2$, $x \in (-\infty, +\infty)$

$\frac{\partial f}{\partial y} = \frac{2yx^2}{(4-y^2)^2}$ cont $y \neq \pm 2$, $x \in (-\infty, +\infty)$





#7) $(x^2+y^2)\, y' = x^2$

$y' = \frac{x^2}{x^2+y^2} = f(x,y)$

$f(x,y)$ is cont for $x \neq 0, y \neq 0$; $(0,0)$

$\frac{\partial f}{\partial y} = \frac{-2yx^2}{(x^2+y^2)}$ continuous for $(0,y) + (x,0)$

#9) $\frac{dy}{dx} = x^3 \cos y = f(x,y)$

$f(x,y)$ is continuous for $x \in (-\infty, +\infty)$, $y \in (-\infty, \infty)$

$\frac{\partial f}{\partial y} = -x^3 \sin y$ " " $x \in (-\infty, +\infty)$, $y \in (-\infty, +\infty)$

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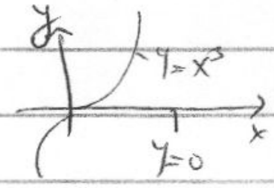
#11) $y' = 3y^{2/3} = 3\sqrt[3]{y^2}$

$\frac{dy}{dx} = 3y^{2/3}$ $\quad y(0) = 0$



$\int \frac{dy}{y^{2/3}} = \int 3\,dx$

$3y^{1/3} = 3x + C$

$y(0) = 0 \implies C = 0$

$y^{1/3} = x \implies \boxed{y = x^3}$

$y = 0$ trivial sol.

$y' = 3y^{2/3} = f(x,y)$

$f(x,y) = 3y^{2/3} = 3\sqrt[3]{y^2}$ is continuous for $y \in (-\infty, +\infty)$

$\frac{\partial f}{\partial y} = y^{-1/3} = \frac{1}{\sqrt[3]{y}}$ " " for $y \neq 0$.

So the E.U. guarantees a unique sol for $y \neq 0$
But $y = 0$ is also a solution that should be added.

#13) $y' = y^3 \longrightarrow \frac{dy}{dx} = y^3 = f(x,y)$

$f(x,y)$ cont. $x \in (-\infty, +\infty)$, $y \in (-\infty, +\infty)$

$\frac{\partial f}{\partial y} = 3y^2$ " "

$\frac{dy}{y^3} = dx$

$\cancel{-\frac{1}{2}y^{-2} = x + C}$

$\cancel{-\frac{1}{2y^2} = x + C}$

$\therefore \exists$ a unique sol. centered @ $x = 0$ which satisfies $y' = y^3$

$\boxed{y = 0}$