STUDY GUIDE FOR FINAL EXAMINATION
MATH 2501

1. Given the parametric equations: $x = \sqrt{t},\ y = 2t^2 + 1$.
   (a) Sketch the curve and show its orientation.
   (b) Find the slope of the curve at $t = 1$.
   (c) Approximate the length of the curve for $1 \le t \le 4$.
   (d) Assume that the parametric equations give the position of an object moving in the coordinate plane as a function of time t. Find the speed of the object at $t = 4$.

2. Given the points $A = (-2, 1, 3)$, $B = (3, 5, -2)$, and $C = (-4, -1, 3)$ in space.

   (a) Find parametric equations for the line containing B and C.

   (b) Find an equation for the plane containing A, B, and C.

   (c) Which of these vectors is perpendicular to the plane containing A, B, and C?

   A. $\langle 5,5,-1\rangle$ B. $\left\langle -\frac{5}{3}, \frac{5}{3}, -\frac{1}{3}\right\rangle$ C. $\left\langle 1, -1, \frac{1}{5}\right\rangle$ D. $\langle 10, 10, 2\rangle$

3. Identify the graph of each equation in space:

   (a) $\dfrac{x^2}{4} + \dfrac{y^2}{16} - \dfrac{z^2}{4} = 1$

   (b) $x^2 + y^2 = 4$

   (c) $4x^2 + y^2 - 4z = 0$

   (d) $z = 2\sqrt{r}$

   (e) $\rho = 2\cos\phi$

4. Given $w = f(x,y,z) = \sqrt{x^2 + y^2 + z^2}$.

   (a) Find $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial^2 f}{\partial y\,\partial x}$, and $\dfrac{\partial^2 f}{\partial x^2}$.

   (b) Identify the graph of the level surface where $w = f(x, y, z) = 2$.

   (c) Use the total differential to approximate $\sqrt{\mathbf{2.04^2 + 1.98^2 + 0.98^2}}$.

5. Given $x^2 + y^2 + 2z^2 = 4$.

   (a) Find $\nabla f(-1, 1, 1)$.

   (b) Find $D_{\mathbf{u}} f(-1,1,1)$ in the direction of $\mathbf{v} = \langle \mathbf{2, -3, 6} \rangle$.

   (c) Find the maximum rate of change of f at $(-1, 1, 1)$.

   (d) Find the equation of the tangent plane to the surface at the point $(-1, 1, 1)$.

   (e) Identify the surface.

6. Given $f(x,y) = x^2 - y^2 - 2x - 6y - 3$.
   (a) Find the critical points and classify using the Second Partials Test.
   (b) Determine the minimum and maximum of f on the closed region R whose boundary is the square with vertices at (0,0), (4,0), (4,4), and (0,4).

7. Find $\mathbf{r}(1)$ if $\mathbf{r}''(t) = -32\mathbf{k}$, $\mathbf{r}'(0) = \mathbf{0}$, and $\mathbf{r}(0) = \mathbf{i} + \mathbf{j}$.

8. Find the acceleration vector of a particle that moves along the curve C described by

$$\mathbf{r}(t) = \ln t\,\mathbf{i} + \frac{1}{t}\mathbf{j} + t^3\mathbf{k}.$$

9. Calculate the speed of a particle whose position is given by

$$\mathbf{r}(t) = \sin t\mathbf{i} + \cos t\mathbf{j} - 16t^2\mathbf{k}$$

at the pont where $t = 1$.

10. Find the maximum and minimum values of $f(x, y, z) = 4x^2 + y^2 + 5z^2$ subject to the constraint $2x + 3y + 4z = 12,\ x \geq 0,\ y \geq 0$.

11. Evaluate $\int_0^1 \int_{2x}^2 e^{y^2}\, dy\, dx$ by reversing the order of integration.

12. Evaluate $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx$ by changing to polar coordinates.

13. Find the volume of the solid bounded by the xy-plane, the cylinder $x^2 + y^2 = 9$, and the paraboloid $z = 2(x^2 + y^2)$.

14. Use a triple integral to find the volume of the solid in the first octant bounded by the coordinate planes, the plane $2x + y = 2$, and the paraboloid $z = 4 - x^2 - y^2$.

15. Use Green's Theorem to evaluate $\oint_C y^2 dx + 6xy\, dy$ where C is the path from (0, 0) to (1, 0) along $y = 0$, from (1, 0) to (1, 1) along $x = 1$, and from (1, 1) to (0, 0) along $y = \sqrt{x}$.

16. Use Stokes' Theorem to evaluate $\oint_C \mathbf{F} \cdot \mathbf{T}\, ds$, where $\mathbf{F}(x, y, z) = (3x - 2y)\mathbf{i} + (4x - 3y)\mathbf{j} + (z + 2y)\mathbf{k}$ and C is the triangle whose vertices are (1, 0, 0), (0, 1, 0), and (0, 0, 1).

Answers

1. (b) 8
   (c) 30.023407
   (d) 16.001953

2. (a) $\begin{cases} x = -7t + 3 \\ y = -6t + 5 \\ z = 5t - 2 \end{cases}$
   (b) $5x - 5y + z = -12$
   (c) B & C

3. (a) Hyperboloid of One Sheet
   (b) Circular cylinder
   (c) Elliptical paraboloid
   (d) Circular Cone
   (e) Sphere

4. (a) $\dfrac{\partial f}{\partial x} = \dfrac{x}{\sqrt{x^2 + y^2 + z^2}}$, $\dfrac{\partial^2 f}{\partial y \partial x} = \dfrac{-xy}{\sqrt{(x^2 + y^2 + z^2)^3}}$, and

   $$\frac{\partial^2 f}{\partial x^2} = \frac{y^2 + z^2}{\sqrt{(x^2 + y^2 + z^2)^3}}$$

   (b) Sphere
   (c) 3.0067

5. (a) $\langle -2, 2, 4 \rangle$
   (b) 2
   (c) $2\sqrt{6}$
   (d) $x - y - 2z = -4$

6. (a) (1, −3, 5) is a saddle point
   (b) maximum is 5 at (4,0); minimum is −44 at (1,4)

7. **i + j** − 16**k**

8. $-\dfrac{1}{t^2}\mathbf{i} + \dfrac{2}{t^3}\mathbf{j} + 6t\mathbf{k}$

9. $5\sqrt{41}$

10. Minimum: $\frac{1320}{121} = f\left(\frac{5}{11}, \frac{30}{11}, \frac{8}{11}\right)$; Maximum: $144 = f(6, 0, 0)$.

11. $\frac{1}{4}(e^4 - 1)$.

12. $\frac{\pi}{2}(e - 1)$

13. 81π

14. $\frac{19}{6}$

15. 1

16. 4