STUDY GUIDE FOR FINAL EXAMINATION
MATH 1502

1. Evaluate the integral below by each of the specified methods.

$$\int \frac{x^3}{\sqrt{4 + x^2}}\, dx$$

   (a) Use the trigonometric substitution $x = 2\tan\theta$.
   (b) Use integration by parts with $u = x^2$.

2. Evaluate each of the indefinite integrals.

   (a) $\int \csc^2 5x\, dx$
   (b) $\int x e^{x^2}\, dx$
   (c) $\int (2 + 3x)^{10}\, dx$
   (d) $\int x^3 \tan x^4\, dx$
   (e) $\int \frac{\sec 3x}{\tan 3x}\, dx$
   (f) $\int (x^2 + 1)^2\, dx$
   (g) $\int \frac{x^2}{x^2 + 4x + 5}\, dx$

3. Evaluate $\int \sec^6 3x\, dx$. (Use the trigonometric identity $1 + \tan^2\theta = \sec^2\theta$.)

4. Use the Trapezoidal Rule with $n = 4$ to approximate

$$\int_0^{2\pi} 2\pi x\sqrt{1 + \cos^2 x}\, dx.$$

5. The table below gives some values of a function L

t	$L(t)$
20	1.0
25	1.2
30	1.0
35	0.9
40	1.0
45	1.1
50	1.3
55	1.4
60	1.3

Approximate $\int_{20}^{60} L(t)\,dt$ using the Trapezoidal Rule and Simpson's Rule.

For problems 6 and 7 do each of the following:

(a) State why the integral is improper.
(b) Express the integral as a limit or limits.
(c) Evaluate the integral and state whether it diverges or converges.

6. $\int_{-7}^{2} (x - 1)^{-2/3}\,dx$

7. $\int_{-\infty}^{0} e^{2x}\,dx$

8. Find the particular solution of the differential equation that satisfies the given initial condition:

$$dT + k(T - 70)dt = 0; \quad T(0) = 140$$

9. Find the sum of the geometric series: $\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^{n}$

10. Approximate the sum of each of these series with error less than 0.0001:

(a) $\sum_{n=1}^{\infty} \frac{1}{n^4}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n^3 + 1}$

11. What information does the n^{th}-Term Test give about each of these series?

(a) $\sum_{n=1}^{\infty} \frac{\sin n}{n}$

(b) $\sum_{n=1}^{\infty} \frac{n^2}{n^2 + 1}$

12. Determine the convergence or divergence of the series using the Limit Comparison Test:

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2 + 1}}$$

Name the series used in the comparison.

13. Determine the convergence or divergence of the series using the Integral Test:

$$\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$$

14. Determine whether the series, $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 3^n}{2^n n^2}$ is absolutely convergent, conditionally convergent, or divergent.

15. What information does the Ratio Test give about this series?

$$\sum_{n=1}^{\infty} \frac{5^{n-1}}{(n+1)!}$$

16. Find the degree 5 Maclaurin polynomial for $f(x) = e^{2x}$. Use it to approximate the value of $e^{0.5}$.

17. Determine the interval of convergence of the power series:

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2 2^{n+1}}.$$

18. Use the Basic List of Taylor Series to determine a power series for
    (a) $f(x) = \cos x^2$
    (b) $f(x) = \dfrac{1}{\sqrt{1 - 16x^2}}$

19. Given the cardioid $r = 1 + \sin\theta$.
    (a) Find the area inside it.
    (b) Find its arc length.

20. Find the area of one leaf of $r = 2\cos 3\theta$.

ANSWERS

1. (a) $\dfrac{(4 + x^2)^{3/2}}{3} - 4\sqrt{4 + x^2} + C$

   (b) $x^2\sqrt{4 + x^2} - \frac{2}{3}(4 + x^2)^{3/2} + C$

   NOTE: These answers are equal. Both simplify to:

   $$\frac{(x^2 - 8)\sqrt{x^2 + 4}}{3} + C$$

2. (a) $-\frac{1}{5}\cot 5x + C$

   (b) $\frac{1}{2}e^{x^2} + C$

   (c) $\frac{1}{33}(2 + 3x)^{11} + C$

   (d) $-\frac{1}{4}\ln|\cos x^4| + C$

   (e) $-\frac{1}{3}\ln|\csc 3x + \cot 3x| + C$

(f) $\frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C$

(g) $x - 2\ln|x^2 + 4x + 5|$

$+ 3\arctan(x + 2) + C$

3. $\frac{1}{15}\tan^5 3x + \frac{2}{3}\tan^3 3x + \frac{1}{3}\tan 3x + C$

4. 149.71

5. Trapezoidal Rule: 38.75
Simpson's Rule: 45.5

6. 9

7. $\frac{1}{2}$

8. $T = 70e^{-kt} + 70$

9. 10

10. (a) 1.08223390683 (S_{15}); error < .000099
(b) 0.585648719085 (S_{21}); error < .000094

11. (a) no information
(b) diverges

12. diverges; $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

13. converges

14. diverges

15. converges

16. $P_5(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{2}{3}x^4 + \frac{4}{15}x^5$;
$e^{0.5} \approx 1.64869791667$

17. [1, 5]

18. (a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$
(b) $\sum_{n=1}^{\infty} \frac{(2n)!\, 2^{2n} x^{2n}}{(n!)^2}$

19. (a) $\frac{3\pi}{2}$
(b) 8

20. $\frac{\pi}{3}$