Dr. ZABDAWI — 1 A

Solution To: Study Guide For Final Examination MATH 1502

1) $I = \int \frac{x^3}{\sqrt{4+x^2}}\, dx$

a) $x = 2\tan\theta \rightarrow \tan\theta = \frac{x}{2} \Rightarrow \theta = \tan^{-1}\frac{x}{2}$, $-\pi/2 < \theta < \pi/2$

$dx = 2\sec^2\theta\, d\theta$

$$I = \int \frac{8\tan^3\theta}{\sqrt{4+4\tan^2\theta}}\, 2\sec^2\theta\, d\theta \quad ; \quad 1+\tan^2\theta = \sec^2\theta$$

$$= 8\int \frac{\tan^3\theta \sec^2\theta\, d\theta}{\sec\theta} = 8\int \tan^3\theta \sec\theta\, d\theta$$

let $u = \sec\theta \Rightarrow du = \sec\theta\tan\theta\, d\theta$

$\tan^2\theta = \sec^2\theta - 1 = u^2 - 1$

$$I = 8\int \tan^2\theta \tan\theta \sec\theta\, d\theta$$

$$= 8\int (u^2-1)\, du = 8\left(\frac{u^3}{3} - u\right) + C$$

$$= 8\left(\frac{\sec^3\theta}{3} - \sec\theta\right) + C$$

$$= 8\left(\frac{(4+x^2)^{3/2}}{8\times 3} - \frac{(4+x^2)^{1/2}}{2}\right) + C$$

$$= \frac{(4+x^2)^{3/2}}{3} - 4(4+x^2)^{1/2} + C$$

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1b) $I = \int \frac{x^3}{\sqrt{4+x^2}}\,dx = \int \frac{x^2 \cdot x\,dx}{\sqrt{4+x^2}}$

Let $u = x^2 \Rightarrow du = 2x\,dx$

$dv = \frac{x\,dx}{\sqrt{4+x^2}}$

Let $z = 4+x^2 \Rightarrow dz = 2x\,dx$

$$\int dv = \int \frac{1}{2}\frac{dz}{z^{1/2}}$$

$$v = \frac{1}{2} z^{1/2} \cdot 2 = z^{1/2} = \sqrt{4+x^2}$$

$$I = x^2\sqrt{4+x^2} - \int \sqrt{4+x^2} \cdot 2x\,dx$$

Let $u = 4+x^2 \Rightarrow du = 2x\,dx$

$$I = x^2\sqrt{4+x^2} - \int u^{1/2}\,du$$

$$= x^2\sqrt{4+x^2} - u^{3/2} \cdot \frac{2}{3} + C$$

$$= x^2\sqrt{4+x^2} - \frac{2}{3}(4+x^2)^{3/2} + C$$

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2a) $\int \csc^2 5x\,dx = -\frac{\cot 5x}{5} + c$

b) $I = \int x e^{x^2}\,dx$ let $u = x^2 \Rightarrow du = 2x\,dx$

$I = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + c = \frac{1}{2}e^{x^2} + c$

c) $\int (2+3x)^{10}\,dx$ let $u = 2+3x \Rightarrow du = 3dx$

$I = \frac{1}{3}\int u^{10}\,du = \frac{1}{3}\frac{u^{11}}{11} + c = \frac{1}{33}(2+3x)^{11} + c$

d) $\int x^3 \tan x^4\,dx$, let $u = x^4 \Rightarrow du = 4x^3 dx$

$I = \frac{1}{4}\int \tan u\,du = \frac{1}{4}\ln|\sec u| + c$

$= \frac{1}{4}\ln|\sec x^4| + c$

$= \frac{1}{4}\ln\left|\frac{1}{\cos x^4}\right| + c$

$= -\frac{1}{4}\ln \cos x^4 + c$

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e) $\int \frac{\sec 3x}{\tan 3x}\,dx = \int \frac{1}{\cos 3x}\cdot\frac{\cos 3x}{\sin 3x}\,dx$

$= \int \frac{1}{\sin 3x}\,dx = \int \csc 3x\,dx$

$= \frac{1}{3}\ln|\csc 3x - \cot 3x| + C$

OR

$= -\frac{1}{3}\ln|\csc 3x + \cot 3x| + C$

f) $\int (x^2+1)^2\,dx = \int (x^4+2x^2+1)\,dx$

$= \frac{x^5}{5} + 2\frac{x^3}{3} + x + C$

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2g) $\displaystyle\int \frac{x^2}{x^2+4x+5}\,dx$

$$\begin{array}{r|l} & \quad 1 \\ x^2+4x+5 & \quad x^2 \\ & \underline{\quad x^2+4x+5} \\ & \quad -4x-5 \end{array}$$

$$I = \int\left(1 - \frac{(4x+5)}{x^2+4x+5}\right)dx$$

$$\frac{4x+5}{x^2+4x+5} = \frac{4x+5}{x^2+4x+4+1} = \frac{4x+5}{(x+2)^2+1}$$

let $u = x+2 \Rightarrow x = u-2$, $du = dx$

$$\frac{4x+5}{(x+2)^2+1} = \frac{4(u-2)+5}{u^2+1} = \frac{4u-3}{u^2+1}$$

$$I = x - \int \frac{4u-3}{u^2+1}\,du$$

$$= x - 4\int\frac{u\,du}{u^2+1} + \int\frac{3}{u^2+1}\,du$$

$$= x - \frac{4}{2}\ln|u^2+1| + 3\tan^{-1}u + C$$

$$= x - 2\ln\left|(x+2)^2+1\right| + 3\tan^{-1}(x+2) + C$$

$$= x - 2\ln\left|x^2+4x+5\right| + 3\tan^{-1}(x+2) + C$$

An extra problem I've just made:

$$\int \frac{x^2}{x^2+5x+4}\,dx$$

$$\begin{array}{r|l} & 1 \\ x^2+5x+4 & x^2 \\ & \underline{x^2+5x+4} \\ & -5x-4 \end{array}$$

$$I = \int\left(1 - \frac{(5x+4)}{x^2+5x+4}\right)dx$$

$$\frac{5x+4}{x^2+5x+4} = \frac{5x+4}{(x+4)(x+1)} = \frac{A}{(x+4)} + \frac{B}{(x+1)}$$

$$A = \left.\frac{5x+4}{(x+1)}\right|_{x=-4} = \frac{-16}{-3} = \frac{16}{3}, \quad A = \frac{16}{3}$$

$$B = \left.\frac{5x+4}{(x+4)}\right|_{x=-1} = \frac{-5+4}{-1+4} = -\frac{1}{3}; \quad B = -1/3$$

$$\therefore I = x - \int\left(\frac{16/3}{(x+4)} - \frac{1/3}{(x+1)}\right)dx$$

$$= x - \frac{16}{3}\ln|x+4| + \frac{1}{3}\ln|x+1| + C$$

$$= x - \frac{16}{3}\ln|x+4| + \frac{1}{3}\ln|x+1| + C$$

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3) $\int \sec^6 3x\,dx = \int \sec^2 3x \left(\sec^4 3x\right) dx$

$$= \int \sec^2 3x \left(1 + \tan^2 3x\right)^2 dx$$

let $u = \tan 3x$

$$du = 3 \sec^2 3x \, dx$$

$$I = \frac{1}{3}\int \left(1+u^2\right)^2 du = \frac{1}{3}\int \left(1 + 2u^2 + u^4\right) du$$

$$= \frac{1}{3}\left(u + 2\frac{u^3}{3} + \frac{u^5}{5}\right) + C$$

$$= \frac{1}{3}\tan 3x + \frac{2}{9}\tan^3 3x + \frac{1}{15}\tan^5 3x + C$$

4) $\int_0^{2\pi} 2\pi x\sqrt{1+\cos^2 x}\,dx \qquad n = 4 \quad , \Delta x = \frac{2\pi - 0}{4} = \frac{\pi}{2}$

T_4 = Trapezoidal with $n=4$ $= \frac{\Delta x}{2}\left[f(0) + 2f(\pi/2) + 2f(\pi) + 2f(3\pi/2) + f(2\pi)\right]$

$$= \frac{\pi}{4}\left[f(0) + 2f(\pi/2) + 2f(\pi) + 2f(3\pi/2) + f(2\pi)\right]$$

$$f(x) = 2\pi x\sqrt{1+\cos^2 x}$$

$$\Rightarrow T_4 = \frac{\pi}{4}\left(0 + 2\times 2\pi\cdot\frac{\pi}{2} + 2\times 2\pi\cdot\pi\sqrt{2} + 2\cdot 2\pi\cdot\frac{3\pi}{2} + 2\pi\times 2\pi\times\sqrt{2}\right)$$

$$= \frac{\pi}{4}\left(2\pi^2 + 4\sqrt{2}\pi^2 + 6\pi^2 + 4\pi^2\sqrt{2}\right) = 149.712$$

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5)

t	$L(t)$
20	1.0
25	1.2
30	1.0
35	0.9
40	1.0
45	1.1
50	1.3
55	1.4
60	1.3

$\Delta t = 5$

$$T_8 = \frac{\Delta t}{2}\left[1 + 2\times 1.2 + 2\times 1.0 + 2\times 0.9 + 2\times 1.0 + 2\times 1.1 + 2\times 1.3 + 2\times 1.4 + 1.3\right]$$

$$= 45.25$$

$$S_8 = \frac{\Delta t}{3}\left[1 + 4\times 1.2 + 2\times 1.0 + 4\times 0.9 + 2\times 1.0 + 4\times 1.1 + 2\times 1.3 + 4\times 1.4 + 1.3\right]$$

$$= 45.5$$

6) $$\int_{-7}^{2} (x-1)^{-2/3}\, dx = \int_{-7}^{2} \frac{1}{(x-1)^{2/3}}\, dx$$

This is an improper integral because the integrand $\frac{1}{(x-1)^{2/3}}$ is undefined at $x=1$.

$$\int_{-7}^{2} \frac{1}{(x-1)^{2/3}}\, dx = \lim_{t \to 1} \int_{-7}^{t} \frac{1}{(x-1)^{2/3}}\, dx + \lim_{t \to 1} \int_{t}^{2} \frac{1}{(x-1)^{2/3}}\, dx$$

$$= \lim_{t \to 1} \left.\frac{(x-1)^{1/3}}{1/3}\right|_{-7}^{t} + \lim_{t \to 1} \left.\frac{(x-1)^{1/3}}{1/3}\right|_{t}^{2}$$

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$$I = \lim_{t\to 1} 3(x-1)^{1/3}\Big|_{-7}^{t} + \lim_{t\to 1} 3(x-1)^{1/3}\Big|_{t}^{2}$$

$$= \lim_{t\to 1} 3\left((t-1)^{1/3} - (-7-1)^{1/3}\right) + \lim_{t\to 1} 3\left((2-1)^{1/3} - (t-1)^{1/3}\right)$$

$$= 6 + 3 = 9 \Rightarrow \text{Converges.}$$

7) $\int_{-\infty}^{0} e^{2x}\,dx$; This is an improper integral of the $-\infty$

$$\int_{-\infty}^{0} e^{2x}dx = \lim_{t\to -\infty} \int_{t}^{0} e^{2x}dx$$

$$= \lim_{t\to -\infty} \frac{e^{2x}}{2}\Big|_{t}^{0}$$

$$= \lim_{t\to -\infty} \frac{1}{2}\left(1 - e^{2t}\right) = 1/2 \Rightarrow \text{Converges.}$$

Skip # 8

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9) $$\sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^n = 5\left(\frac{2}{3}\right) + 5\left(\frac{2}{3}\right)^2 + 5\left(\frac{2}{3}\right)^3 + \cdots$$

$$= 5\left(\frac{2}{3} + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^3 + \left(\frac{2}{3}\right)^4 + \cdots\right)$$

This is a geometric series with $r = \frac{2}{3} < 1$

$\Rightarrow$ Series converge to $5\left(\frac{a}{1-r}\right) = 5\left(\frac{2/3}{1-2/3}\right)$

$$\Rightarrow \sum_{n=1}^{\infty} 5\left(\frac{2}{3}\right)^n = 5\left(\frac{2}{3-2}\right) = 10.$$

10) a) $$\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{1}{1} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \cdots$$

From the integral test

$$\int_{n+1}^{\infty} f(x)\,dx \le R_n(x) \le \int_{n}^{\infty} f(x)\,dx$$

$$\int_n^{\infty} \frac{1}{x^4}\,dx = \lim_{t\to\infty} \int_n^t x^{-4}\,dx = \lim_{t\to\infty} \frac{x^{-3}}{-3}\Big|_n^t = \lim_{t\to\infty} -\frac{1}{3}\left(\frac{1}{t^3} - \frac{1}{n^3}\right)$$

$$= \frac{1}{3n^3}$$

Now $\frac{1}{3n^3} \le .0001 = 10^{-4}$

$n=15$

Any 15 terms

$$3n^3 \ge \frac{1}{10^{-4}} = 10^4$$

$$n^3 \ge \frac{10^4}{3} \Rightarrow n \ge \sqrt[3]{\frac{10^4}{3}} = 14.938 \approx 15$$

$$\sum_{n=1}^{\infty} \frac{1}{n^4} \cong \frac{1}{1} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \frac{1}{6^4} + \frac{1}{7^4} + \frac{1}{8^4} + \frac{1}{9^4} + \frac{1}{10^4} + \frac{1}{11^4}$$

$$+ \frac{1}{12^4} + \frac{1}{13^4} + \frac{1}{14^4} + \frac{1}{15^4} = 1.082233907$$

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10b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n^3+1)}$ This is an alternating series

$$b_n = \frac{1}{n^3+1}$$

$$|R_n| \leq b_{n+1}$$

$$\Rightarrow \frac{1}{(n+1)^3+1} \leq 0.0001 = 10^{-4}$$

$$(n+1)^3+1 \geq \frac{1}{10^{-4}} = 10^4$$

$$(n+1)^3 \geq 10^4-1$$

$$n \geq (10^4-1)^{1/3}-1 = 20.543 \Rightarrow n=21$$

$\Rightarrow$ take ~~21~~ Terms.
Carry 22 Terms

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(n^3+1)} \approx S_{21} = \frac{1}{(0+1)} - \frac{1}{(1^3+1)} + \frac{1}{(2^3+1)} - \frac{1}{(3^3+1)} + \frac{1}{(4^3+1)}$$
$$- \frac{1}{(5^3+1)} + \frac{1}{(6^3+1)} - \frac{1}{(7^3+1)} + \frac{1}{(8^3+1)}$$
$$- \frac{1}{(9^3+1)} + \frac{1}{(10^3+1)} - \frac{1}{(11^3+1)} + \frac{1}{(12^3+1)}$$
$$- \frac{1}{(13^3+1)} + \frac{1}{(14^3+1)} - \frac{1}{(15^3+1)} + \frac{1}{(16^3+1)}$$
$$- \frac{1}{(17^3+1)} + \frac{1}{(18^3+1)} - \frac{1}{(19^3+1)} + \frac{1}{(20^3+1)}$$
$$- \frac{1}{(21^3+1)}$$

$n=21$
$\Rightarrow$ 22 Terms

$$= 0.5856487191$$

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11) a) $\sum_{n=1}^{\infty} \frac{\sin n}{n}$

$\lim_{n\to\infty} \frac{\sin n}{n} = \frac{\pm 1}{\infty} = 0$

$\Rightarrow$ No information can be drawn

Because if $\lim_{n\to\infty} a_n = 0 \Rightarrow$ Series could converge or diverge

b) $\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$; $\lim_{n\to\infty} \frac{n^2}{n^2+1} = 1 \neq 0$

$\therefore$ Series Diverges.

12) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}}$ Take the dominant term on top and the dominant term on bottom

$\Rightarrow$ Compare with $\sum_{n=1}^{\infty} \frac{n^{1/2}}{n} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

Now $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$; This is a P series with $P = 1/2 < 1$

$\Rightarrow$ Series Diverges.

Now use the limit comparison test.

$$\lim_{n\to\infty} \left| \frac{\sqrt{n}}{\sqrt{n^2+1}} \cdot \sqrt{n} \right| = \lim_{n\to\infty} \left| \frac{n}{\sqrt{n^2+1}} \right| = \lim_{n\to\infty} \left| \frac{1}{\sqrt{1+1/n^2}} \right|$$

$= 1 > 0 \Rightarrow \sum_{n=1}^{\infty} \frac{\sqrt{n}}{\sqrt{n^2+1}}$ Diverges by the limit comparison test

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13) $\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$

Integral Test $\int_1^{\infty} \frac{x}{e^{x^2}} dx$

let $u = x^2 \Rightarrow du = 2x\,dx \rightarrow x\,dx = \frac{1}{2} du$

$$\int_1^{\infty} \frac{x}{e^{x^2}} dx = \lim_{t \to \infty} \int_1^t \frac{1}{2} \frac{du}{e^u}$$

$$= \lim_{t \to \infty} \left(\frac{1}{2} \frac{e^{-u}}{-1} \right) \Big|_1^t = \lim_{t \to \infty} -\frac{1}{2} \left(+e^{-t} - e^{-1} \right)$$

$$= \frac{1}{2} e^{-1} = \frac{1}{2e}$$

$\Rightarrow$ Series Converges.

14) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3^n}{2^n n^2}$ Use the ratio test

$$\lim_{n \to \infty} \left| \frac{3^{n+1}}{2^{n+1} (n+1)^2} \cdot \frac{2^n n^2}{3^n} \right| = \lim_{n \to \infty} \left| \frac{3}{2} \cdot \frac{n^2}{(n+1)^2} \right|$$

$$= \lim_{n \to \infty} \left| \frac{3}{2} \left(\frac{n}{n+1} \right)^2 \right| = \frac{3}{2} > 1$$

$\Rightarrow$ Series Diverges.

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15) $\sum_{n=1}^{\infty} \frac{5^{n-1}}{(n+1)!}$

$$\lim_{n\to\infty} \left| \frac{5^{(n+1)-1}}{(n+1+1)!} \cdot \frac{(n+1)!}{5^{n-1}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{5^n}{(n+2)!} \frac{(n+1)!}{5^{n-1}} \right| = \lim_{n\to\infty} \left| \frac{5}{(n+1)} \right| = 0 < 1$$

$\Rightarrow$ Series Converges

16) $f(x) = e^{2x}$ Maclaurin Series

$$f(x) = \sum_{n=0}^{\infty} f^n(0) \frac{x^n}{n!}$$

$f(x) = e^{2x}$ $\quad f(0) = 1$

$f'(x) = 2e^{2x}$ $\quad f'(0) = 2$

$f''(x) = 4e^{2x}$ $\quad f''(0) = 4$

$f'''(x) = 8e^{2x}$ $\quad f'''(0) = 8$

$f^{iv}(x) = 16e^{2x}$ $\quad f^{iv}(0) = 8$

$f^n(x) = 2^n e^{2x}$ $\quad f^n(0) = 2^n$

$\Rightarrow e^{2x} = \sum_{n=0}^{\infty} 2^n \frac{x^n}{n!}$ So for $n = 5$, $x = \frac{1}{4} = 0,25$

$$e^{2x} = e^{0,5} \approx 2^0 \frac{(0,25)^0}{0!} + 2^1 \frac{(0,25)^1}{1!} + 2^2 \frac{(0,25)^2}{2!} + 2^3 \frac{(0,25)^3}{3!}$$

$$+ 2^4 \frac{(0,25)^4}{4!} + 2^5 \frac{(0,25)^5}{5!}$$

16) Continue Dr. BABDAWI

Again $e^{2x} = \sum_{n=0}^{\infty} 2^n \frac{x^n}{n!}$

Take $n=5$ as requested in the problem.

for $x = \frac{1}{4} \Rightarrow e^{2x} = e^{0.5}$

$$e^{0.5} \cong \frac{2^0(0.25)^0}{0!} + \frac{2^1(0.25)^1}{1!} + \frac{2^2(0.25)^2}{2!} + \frac{2^3(0.25)^3}{3!}$$
$$+ \frac{2^4(0.25)^4}{4!} + \frac{2^5(0.25)^5}{5!}$$

$$e^{0.5} \simeq 1.648697917$$

17) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2 2^{n+1}}$

Use the Ratio test $\Rightarrow \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$

$$\Rightarrow \lim_{n\to\infty} \left| \frac{(x-3)^{n+1}}{(n+1)^2 2^{n+1+1}} \cdot \frac{n^2 2^{n+1}}{(x-3)^n} \right| < 1$$

$$\Rightarrow \lim_{n\to\infty} \left| \frac{(x-3)}{2} \cdot \left(\frac{n}{(n+1)}\right)^2 \right| < 1$$

$$\Rightarrow \left| \frac{x-3}{2} \right| < 1 \Rightarrow |x-3| < 2$$

$$\Rightarrow -2 < x-3 < 2$$

$$1 < x < 5$$

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Now we have to check the end points.

@ $x=1$ the series becomes $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2 2^{n+1}}$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n^2 2^{n+1}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{2n^2}$$

lets look at $\sum_{n=1}^{\infty} \frac{1}{2n^2}$ P series $p=2>1$ $\Rightarrow$ Convergent

So $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^2}$ is absolutely convergent and therefore convergent.

@ $x=5$ the series becomes $\sum_{n=1}^{\infty} \frac{2^n}{n^2 2^{n+1}} = \sum_{n=1}^{\infty} \frac{1}{2n^2}$

This is a P-series with $P=2>1 \Rightarrow$ Convergent.

$\therefore$ the Interval of Convergence is $I.C = [1,5]$.

18) a) $f(x) = \cos x^2$

Here I have to give you that,

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{2n!}$$

$$\Rightarrow \cos x^2 = \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{(2n)!}$$

Skip 18b)

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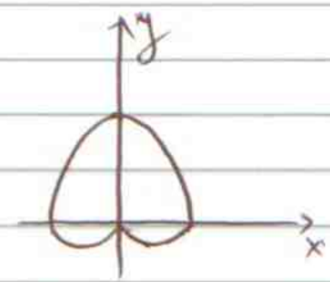
19) $r = 1 + \sin\theta$

θ	r
0	1
$\pi/2$	2
π	1
$3\pi/2$	0
2π	1



a) Find the area inside the Cardioid.

$$A = 2\int_{-\pi/2}^{\pi/2} \frac{1}{2} r^2 \, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} (1+\sin\theta)^2 \, d\theta = \int_{-\pi/2}^{\pi/2} (1 + 2\sin\theta + \sin^2\theta) \, d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \left(1 + 2\sin\theta + \left(\frac{1-\cos 2\theta}{2}\right)\right) d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\pi/2} \left(\frac{3}{2} + 2\sin\theta - \frac{\cos 2\theta}{2}\right) d\theta$$

$$= \left(\frac{3}{2}\theta - 2\cos\theta - \frac{1}{2}\frac{\sin 2\theta}{2}\right)\Bigg|_{-\pi/2}^{\pi/2}$$

$$= \left(\frac{3\pi}{4} - 0 - 0\right) - \left(-\frac{3\pi}{4} - 0 - 0\right) = \frac{3\pi}{2} \text{ Square Units.}$$

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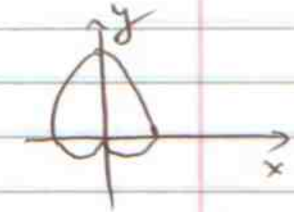
19b) Find the arc length of the Cardioid



$$L = 2\int_{-\pi/2}^{\pi/2} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$r = 1 + \sin\theta \implies \frac{dr}{d\theta} = \cos\theta$

$$r^2 + \left(\frac{dr}{d\theta}\right)^2 = (1+\sin\theta)^2 + \cos^2\theta$$
$$= 1 + 2\sin\theta + \sin^2\theta + \cos^2\theta = 2 + 2\sin\theta$$
$$= 2(1+\sin\theta)$$

$$L = 2\int_{-\pi/2}^{\pi/2} \sqrt{2(1+\sin\theta)}\, d\theta = 2\sqrt{2}\int_{-\pi/2}^{\pi/2} \sqrt{1+\sin\theta}\, d\theta$$

$$= 2\sqrt{2}\int_{-\pi/2}^{\pi/2} \frac{\sqrt{1-\sin^2\theta}}{\sqrt{1-\sin\theta}}\, d\theta \quad ; \ 1-\sin^2\theta = \cos^2\theta, \ \sqrt{\cos^2\theta} = |\cos\theta|$$

$$= 2\sqrt{2}\int_{-\pi/2}^{\pi/2} \frac{|\cos\theta|}{\sqrt{1-\sin\theta}}\, d\theta, \quad \cos\theta \text{ is (+)ve between } -\pi/2 \ \& \ \pi/2$$

$$L = 2\sqrt{2}\int_{-\pi/2}^{\pi/2} \frac{\cos\theta\, d\theta}{\sqrt{1-\sin\theta}}$$

let $u = 1-\sin\theta \implies du = -\cos\theta\, d\theta$

$$L = 2\sqrt{2}\int_{2}^{0} \frac{-du}{u^{1/2}} = 2\sqrt{2}\int_{0}^{2} \frac{du}{u^{1/2}} = 2\sqrt{2}\int_{0}^{2} u^{-1/2}\, du$$

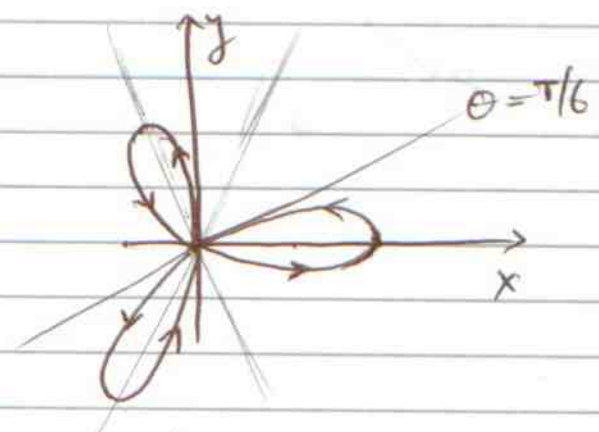
$$= 2\sqrt{2}\, u^{1/2} \cdot 2 \Big|_0^2 = 4\sqrt{2}\left(2^{1/2} - 0\right) = 4\sqrt{2}\sqrt{2} = 8 \text{ Square Units}$$

Dr ZABDAWI

20) Find the area of one leaf of $r = 2\cos 3\theta$

Period $= \frac{2\pi}{3}$, $\frac{P}{4} = \frac{2\pi}{3} \cdot \frac{1}{4} = \frac{\pi}{6}$

θ	r
0	2
$\frac{\pi}{6}$	0
$\frac{\pi}{3}$	-2
$\frac{\pi}{2}$	0
$2\pi/3$	2
$\frac{5\pi}{6}$	0
π	-2



$$\text{Area} = 2\int_0^{\pi/6} \frac{1}{2} r^2 \, d\theta$$

$$= \int_0^{\pi/6} 4\cos^2 3\theta \, d\theta$$

$$= 4\int_0^{\pi/6} \left(\frac{1+\cos 6\theta}{2}\right) d\theta$$

$$= 2\int_0^{\pi/6} (1+\cos 6\theta)\, d\theta$$

$$= 2\left(\theta + \frac{\sin 6\theta}{6}\right)\Big|_0^{\pi/6}$$

$$= 2\left[\left(\frac{\pi}{6}+0\right) - (0+0)\right] = \frac{\pi}{3} \text{ Square Units}$$