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Series

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Strategy	Form and/or conditions	How to test for convergence and divergence
Geometric Series Test	$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \ldots$	$S_n = \frac{a(1-r^n)}{1-r}$ If $\|r\| < 1$, the series will converge to $\frac{a}{1-r}$ If $\|r\| \geq 1$, the series will diverge
Test for Divergence	$\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n = \pm\infty$	If $\lim_{n\to\infty} a_n \neq 0$, or $\lim_{n\to\infty} a_n = \pm\infty$, then the series will diverge (Note: the opposite is not true. If $\lim_{n\to\infty} a_n = 0$, then the series may converge or diverge)
Integral Test	f is a continuous, positive, decreasing function on $[1,\infty)$ and $a_n = f(n)$ $\int_{n+1}^{\infty} f(x)dx \leq R_n \leq \int_{n}^{\infty} f(x)dx$ $s_n + \int_{n+1}^{\infty} f(x)dx \leq s \leq s_n + \int_{n}^{\infty} f(x)dx$	If $\int_{1}^{\infty} f(x)dx$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent. If $\int_{1}^{\infty} f(x)dx$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	If $p > 1$, the series is convergent. If $p \leq 1$ the series is divergent.
Comparison Test	$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms	If $\sum_{n=1}^{\infty} b_n$ is convergent and $a_n \leq b_n \ \forall n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ is also convergent. If $\sum_{n=1}^{\infty} b_n$ is divergent and $a_n \geq b_n \ \forall n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ is also divergent.
Limit Comparison Test	$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms	If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, where c is a finite number > 0, then either both series converge or both series diverge. If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$, and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.
Alternating Series Test	$\sum (-1)^{n-1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 \ldots$ $b_n > 0$	If $b_{n+1} \leq b_n$ for $\forall n \geq 1$, and $\lim_{n\to\infty} b_n = 0$, then the series converges. $\|R_n\| \leq b_{n+1}$
Ratio Test	$\lim_{n\to\infty} \left\| \frac{a_{n+1}}{a_n} \right\| = L$ (Use when factorials or numbers to the *n*th power are used)	If L < 1, the series is absolutely convergent and hence convergent. If L > 1 or L= ∞, the series is divergent. If L = 1, no conclusion can be drawn.
Root Test	$\lim_{n\to\infty} \sqrt[n]{\|a_n\|} = L$ (Use when the whole argument is raised to the *n*th power)	If L < 1, the series is absolutely convergent and hence convergent. If L > 1 or L= ∞, the series is divergent. If L = 1, no conclusion can be drawn.