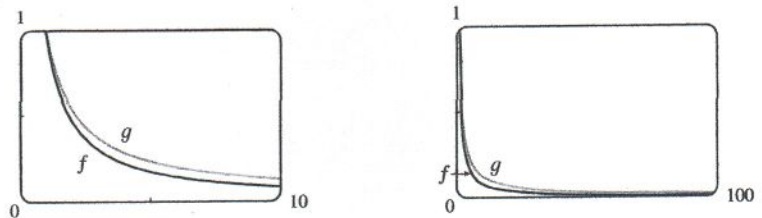


8.8 Improper Integrals

1. (a) Since $\int_1^\infty x^4 e^{-x^4} dx$ has an infinite interval of integration, it is an improper integral of Type I.
 (b) Since $y = \sec x$ has an infinite discontinuity at $x = \frac{\pi}{2}$, $\int_0^{\pi/2} \sec x dx$ is a Type II improper integral.
 (c) Since $y = \frac{x}{(x-2)(x-3)}$ has an infinite discontinuity at $x = 2$, $\int_0^2 \frac{x}{x^2 - 5x + 6} dx$ is a Type II improper integral.
 (d) Since $\int_{-\infty}^0 \frac{1}{x^2 + 5} dx$ has an infinite interval of integration, it is an improper integral of Type I.
2. (a) Since $y = 1/(2x - 1)$ is defined and continuous on $[1, 2]$, the integral is proper.
 (b) Since $y = \frac{1}{2x - 1}$ has an infinite discontinuity at $x = \frac{1}{2}$, $\int_0^1 \frac{1}{2x - 1} dx$ is a Type II improper integral.
 (c) Since $\int_{-\infty}^\infty \frac{\sin x}{1 + x^2} dx$ has an infinite interval of integration, it is an improper integral of Type I.
 (d) Since $y = \ln(x - 1)$ has an infinite discontinuity at $x = 1$, $\int_1^2 \ln(x - 1) dx$ is a Type II improper integral.
3. The area under the graph of $y = 1/x^3 = x^{-3}$ between $x = 1$ and $x = t$ is
 $A(t) = \int_1^t x^{-3} dx = [-\frac{1}{2}x^{-2}]_1^t = -\frac{1}{2}t^{-2} - (-\frac{1}{2}) = \frac{1}{2} - 1/(2t^2)$. So the area for $1 \leq x \leq 10$ is
 $A(10) = 0.5 - 0.005 = 0.495$, the area for $1 \leq x \leq 100$ is $A(100) = 0.5 - 0.00005 = 0.49995$, and the area for
 $1 \leq x \leq 1000$ is $A(1000) = 0.5 - 0.0000005 = 0.4999995$. The total area under the curve for $x \geq 1$ is
 $\lim_{t \rightarrow \infty} A(t) = \lim_{t \rightarrow \infty} [\frac{1}{2} - 1/(2t^2)] = \frac{1}{2}$.

4. (a)

(b) The area under the graph of f from $x = 1$ to $x = t$ is

$$F(t) = \int_1^t f(x) dx = \int_1^t x^{-1.1} dx = \left[-\frac{1}{0.1}x^{-0.1}\right]_1^t \\ = -10(t^{-0.1} - 1) = 10(1 - t^{-0.1})$$

and the area under the graph of g is

$$G(t) = \int_1^t g(x) dx = \int_1^t x^{-0.9} dx = \left[\frac{1}{0.1}x^{0.1}\right]_1^t \\ = 10(t^{0.1} - 1)$$

t	$F(t)$	$G(t)$
10	2.06	2.59
100	3.69	5.85
10^4	6.02	15.12
10^6	7.49	29.81
10^{10}	9	90
10^{20}	9.9	990

(c) The total area under the graph of f is $\lim_{t \rightarrow \infty} F(t) = \lim_{t \rightarrow \infty} 10(1 - t^{-0.1}) = 10$.The total area under the graph of g does not exist, since $\lim_{t \rightarrow \infty} G(t) = \lim_{t \rightarrow \infty} 10(t^{0.1} - 1) = \infty$.

$$5. I = \int_1^{\infty} \frac{1}{(3x+1)^2} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{1}{(3x+1)^2} dx. \text{ Now}$$

$$\int \frac{1}{(3x+1)^2} dx = \frac{1}{3} \int \frac{1}{u^2} du \quad [u = 3x+1, du = 3 dx]$$

$$= -\frac{1}{3u} + C = -\frac{1}{3(3x+1)} + C,$$

$$\text{so } I = \lim_{t \rightarrow \infty} \left[-\frac{1}{3(3x+1)} \right]_1^t = \lim_{t \rightarrow \infty} \left[-\frac{1}{3(3t+1)} + \frac{1}{12} \right] = 0 + \frac{1}{12} = \frac{1}{12}. \text{ Convergent}$$

$$6. \int_{-\infty}^0 \frac{1}{2x-5} dx = \lim_{t \rightarrow -\infty} \int_t^0 \frac{1}{2x-5} dx = \lim_{t \rightarrow -\infty} \left[\frac{1}{2} \ln |2x-5| \right]_t^0 = \lim_{t \rightarrow -\infty} \left[\frac{1}{2} \ln 5 - \frac{1}{2} \ln |2t-5| \right] = -\infty.$$

Divergent

$$7. \int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}} dw = \lim_{t \rightarrow -\infty} \int_t^{-1} \frac{1}{\sqrt{2-w}} dw = \lim_{t \rightarrow -\infty} \left[-2\sqrt{2-w} \right]_t^{-1} \quad [u = 2-w, du = -dw]$$

$$= \lim_{t \rightarrow -\infty} \left[-2\sqrt{3} + 2\sqrt{2-t} \right] = \infty. \text{ Divergent}$$

$$8. \int_0^{\infty} \frac{x}{(x^2+2)^2} dx = \lim_{t \rightarrow \infty} \int_0^t \frac{x}{(x^2+2)^2} dx = \lim_{t \rightarrow \infty} \frac{1}{2} \left[\frac{-1}{x^2+2} \right]_0^t = \frac{1}{2} \lim_{t \rightarrow \infty} \left(\frac{-1}{t^2+2} + \frac{1}{2} \right)$$

$$= \frac{1}{2} \left(0 + \frac{1}{2} \right) = \frac{1}{4}. \text{ Convergent}$$

$$9. \int_4^{\infty} e^{-y/2} dy = \lim_{t \rightarrow \infty} \int_4^t e^{-y/2} dy = \lim_{t \rightarrow \infty} \left[-2e^{-y/2} \right]_4^t = \lim_{t \rightarrow \infty} (-2e^{-t/2} + 2e^{-2}) = 0 + 2e^{-2} = 2e^{-2}.$$

Convergent

$$10. \int_{-\infty}^{-1} e^{-2t} dt = \lim_{x \rightarrow -\infty} \int_x^{-1} e^{-2t} dt = \lim_{x \rightarrow -\infty} \left[-\frac{1}{2} e^{-2t} \right]_x^{-1} = \lim_{x \rightarrow -\infty} \left[-\frac{1}{2} e^2 + \frac{1}{2} e^{-2x} \right] = \infty. \text{ Divergent}$$

$$11. \int_{-\infty}^{\infty} \frac{x dx}{1+x^2} = \int_{-\infty}^0 \frac{x dx}{1+x^2} + \int_0^{\infty} \frac{x dx}{1+x^2} \text{ and}$$

$$\int_0^{\infty} \frac{x dx}{1+x^2} = \lim_{t \rightarrow \infty} \left[\frac{1}{2} \ln(1+x^2) \right]_0^t = \lim_{t \rightarrow \infty} \left[0 - \frac{1}{2} \ln(1+t^2) \right] = -\infty. \text{ Divergent}$$

$$12. I = \int_{-\infty}^{\infty} (2-v^4) dv = I_1 + I_2 = \int_{-\infty}^0 (2-v^4) dv + \int_0^{\infty} (2-v^4) dv, \text{ but}$$

$$I_1 = \lim_{t \rightarrow -\infty} \left[2v - \frac{1}{5}v^5 \right]_t^0 = \lim_{t \rightarrow -\infty} \left(-2t + \frac{1}{5}t^5 \right) = -\infty. \text{ Since } I_1 \text{ is divergent, } I \text{ is divergent, and there is no need to evaluate } I_2. \text{ Divergent}$$

$$13. \int_{-\infty}^{\infty} x e^{-x^2} dx = \int_{-\infty}^0 x e^{-x^2} dx + \int_0^{\infty} x e^{-x^2} dx.$$

$$\int_{-\infty}^0 x e^{-x^2} dx = \lim_{t \rightarrow -\infty} \left(-\frac{1}{2} \right) \left[e^{-x^2} \right]_t^0 = \lim_{t \rightarrow -\infty} \left(-\frac{1}{2} \right) (1 - e^{-t^2}) = -\frac{1}{2} \cdot 1 = -\frac{1}{2}, \text{ and}$$

$$\int_0^{\infty} x e^{-x^2} dx = \lim_{t \rightarrow \infty} \left(-\frac{1}{2} \right) \left[e^{-x^2} \right]_0^t = \lim_{t \rightarrow \infty} \left(-\frac{1}{2} \right) (e^{-t^2} - 1) = -\frac{1}{2} \cdot (-1) = \frac{1}{2}.$$

$$\text{Therefore, } \int_{-\infty}^{\infty} x e^{-x^2} dx = -\frac{1}{2} + \frac{1}{2} = 0. \text{ Convergent}$$

$$14. \int_{-\infty}^{\infty} x^2 e^{-x^3} dx = \int_{-\infty}^0 x^2 e^{-x^3} dx + \int_0^{\infty} x^2 e^{-x^3} dx, \text{ and}$$

$$\int_{-\infty}^0 x^2 e^{-x^3} dx = \lim_{t \rightarrow -\infty} \left[-\frac{1}{3} e^{-x^3} \right]_t^0 = -\frac{1}{3} + \frac{1}{3} \left(\lim_{t \rightarrow -\infty} e^{-t^3} \right) = \infty. \text{ Divergent}$$

$$15. \int_{2\pi}^{\infty} \sin \theta d\theta = \lim_{t \rightarrow \infty} \int_{2\pi}^t \sin \theta d\theta = \lim_{t \rightarrow \infty} [-\cos \theta]_{2\pi}^t = \lim_{t \rightarrow \infty} (-\cos t + 1). \text{ This limit does not exist, so the integral}$$

is divergent. Divergent

$$16. \int_0^{\infty} \cos^2 \alpha \, d\alpha = \lim_{t \rightarrow \infty} \int_0^t \frac{1}{2} (1 + \cos 2\alpha) \, d\alpha = \lim_{t \rightarrow \infty} \left[\frac{1}{2} \alpha + \frac{1}{4} \sin 2\alpha \right]_0^t = \lim_{t \rightarrow \infty} \left[\frac{1}{2} t + \frac{1}{4} \sin 2t \right] = \infty \text{ since } \left| \frac{1}{4} \sin 2t \right| \leq \frac{1}{4} \text{ for all } t, \text{ but } \frac{1}{2} t \rightarrow \infty \text{ as } t \rightarrow \infty. \text{ Divergent}$$

$$17. \int_1^{\infty} \frac{x+1}{x^2+2x} \, dx = \lim_{t \rightarrow \infty} \int_1^t \frac{\frac{1}{2}(2x+2)}{x^2+2x} \, dx = \frac{1}{2} \lim_{t \rightarrow \infty} [\ln(x^2+2x)]_1^t = \frac{1}{2} \lim_{t \rightarrow \infty} [\ln(t^2+2t) - \ln 3] = \infty. \text{ Divergent}$$

$$18. \int_0^{\infty} \frac{dz}{z^2+3z+2} = \lim_{t \rightarrow \infty} \int_0^t \left[\frac{1}{z+1} - \frac{1}{z+2} \right] dz = \lim_{t \rightarrow \infty} \left[\ln \left(\frac{z+1}{z+2} \right) \right]_0^t \\ = \lim_{t \rightarrow \infty} \left[\ln \left(\frac{t+1}{t+2} \right) - \ln \left(\frac{1}{2} \right) \right] = \ln 1 + \ln 2 = \ln 2. \text{ Convergent}$$

$$19. \int_0^{\infty} s e^{-5s} \, ds = \lim_{t \rightarrow \infty} \int_0^t s e^{-5s} \, ds = \lim_{t \rightarrow \infty} \left[-\frac{1}{5} s e^{-5s} - \frac{1}{25} e^{-5s} \right]_0^t \quad \left[\begin{array}{l} \text{by integration by} \\ \text{parts with } u = s \end{array} \right] \\ = \lim_{t \rightarrow \infty} \left(-\frac{1}{5} t e^{-5t} - \frac{1}{25} e^{-5t} + \frac{1}{25} \right) = 0 - 0 + \frac{1}{25} \quad \text{[by l'Hospital's Rule]} \\ = \frac{1}{25}. \text{ Convergent}$$

$$20. \int_{-\infty}^6 r e^{r/3} \, dr = \lim_{t \rightarrow -\infty} \int_t^6 r e^{r/3} \, dr = \lim_{t \rightarrow -\infty} \left[3r e^{r/3} - 9e^{r/3} \right]_t^6 \quad \left[\begin{array}{l} \text{by integration by} \\ \text{parts with } u = r \end{array} \right] \\ = \lim_{t \rightarrow -\infty} (18e^2 - 9e^2 - 3t e^{t/3} + 9e^{t/3}) = 9e^2 - 0 + 0 \quad \text{[by l'Hospital's Rule]} \\ = 9e^2. \text{ Convergent}$$

$$21. \int_1^{\infty} \frac{\ln x}{x} \, dx = \lim_{t \rightarrow \infty} \left[\frac{(\ln x)^2}{2} \right]_1^t \quad \text{(by substitution with } u = \ln x, du = dx/x) = \lim_{t \rightarrow \infty} \frac{(\ln t)^2}{2} = \infty. \text{ Divergent}$$

$$22. \int_{-\infty}^{\infty} e^{-|x|} \, dx = \int_{-\infty}^0 e^x \, dx + \int_0^{\infty} e^{-x} \, dx, \int_{-\infty}^0 e^x \, dx = \lim_{t \rightarrow -\infty} [e^x]_t^0 = \lim_{t \rightarrow -\infty} (1 - e^t) = 1, \text{ and} \\ \int_0^{\infty} e^{-x} \, dx = \lim_{t \rightarrow \infty} [-e^{-x}]_0^t = \lim_{t \rightarrow \infty} (1 - e^{-t}) = 1. \text{ Therefore, } \int_{-\infty}^{\infty} e^{-|x|} \, dx = 1 + 1 = 2. \text{ Convergent}$$

$$23. \int_{-\infty}^{\infty} \frac{x^2}{9+x^6} \, dx = \int_{-\infty}^0 \frac{x^2}{9+x^6} \, dx + \int_0^{\infty} \frac{x^2}{9+x^6} \, dx = 2 \int_0^{\infty} \frac{x^2}{9+x^6} \, dx \quad \text{[since the integrand is even].}$$

$$\text{Now } \int \frac{x^2 \, dx}{9+x^6} \quad \left[\begin{array}{l} u = x^3 \\ du = 3x^2 \, dx \end{array} \right] = \int \frac{\frac{1}{3} du}{9+u^2} \quad \left[\begin{array}{l} u = 3v \\ du = 3 \, dv \end{array} \right] = \int \frac{\frac{1}{3}(3 \, dv)}{9+9v^2} = \frac{1}{9} \int \frac{dv}{1+v^2} \\ = \frac{1}{9} \tan^{-1} v + C = \frac{1}{9} \tan^{-1} \left(\frac{u}{3} \right) + C = \frac{1}{9} \tan^{-1} \left(\frac{x^3}{3} \right) + C,$$

$$\text{so } 2 \int_0^{\infty} \frac{x^2}{9+x^6} \, dx = 2 \lim_{t \rightarrow \infty} \int_0^t \frac{x^2}{9+x^6} \, dx = 2 \lim_{t \rightarrow \infty} \left[\frac{1}{9} \tan^{-1} \left(\frac{x^3}{3} \right) \right]_0^t \\ = 2 \lim_{t \rightarrow \infty} \frac{1}{9} \tan^{-1} \left(\frac{t^3}{3} \right) = \frac{2}{9} \cdot \frac{\pi}{2} = \frac{\pi}{9}. \text{ Convergent}$$

$$24. \text{ Integrate by parts with } u = \ln x, dv = dx/x^3 \Rightarrow du = dx/x, v = -1/(2x^2).$$

$$\int_1^{\infty} \frac{\ln x}{x^3} \, dx = \lim_{t \rightarrow \infty} \int_1^t \frac{\ln x}{x^3} \, dx = \lim_{t \rightarrow \infty} \left(\left[-\frac{1}{2x^2} \ln x \right]_1^t + \frac{1}{2} \int_1^t \frac{1}{x^3} \, dx \right) \\ = \lim_{t \rightarrow \infty} \left(-\frac{1}{2} \frac{\ln t}{t^2} + 0 - \frac{1}{4t^2} + \frac{1}{4} \right) = \frac{1}{4}$$

$$\text{since } \lim_{t \rightarrow \infty} \frac{\ln t}{t^2} = \lim_{t \rightarrow \infty} \frac{1/t}{2t} = \lim_{t \rightarrow \infty} \frac{1}{2t^2} = 0. \text{ Convergent}$$

25. Integrate by parts with $u = \ln x$, $dv = dx/x^2 \Rightarrow du = dx/x$, $v = -1/x$.

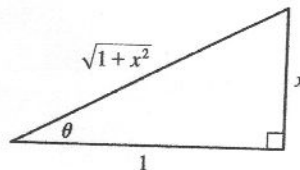
$$\int_1^{\infty} \frac{\ln x}{x^2} dx = \lim_{t \rightarrow \infty} \int_1^t \frac{\ln x}{x^2} dx = \lim_{t \rightarrow \infty} \left[-\frac{\ln x}{x} - \frac{1}{x} \right]_1^t = \lim_{t \rightarrow \infty} \left(-\frac{\ln t}{t} - \frac{1}{t} + 0 + 1 \right) \\ = -0 - 0 + 0 + 1 = 1$$

since $\lim_{t \rightarrow \infty} \frac{\ln t}{t} \stackrel{H}{=} \lim_{t \rightarrow \infty} \frac{1/t}{1} = 0$. Convergent

26. $\int_0^{\infty} \frac{x \arctan x}{(1+x^2)^2} dx = \lim_{t \rightarrow \infty} \int_0^t \frac{x \arctan x}{(1+x^2)^2} dx$. Let $u = \arctan x$, $dv = \frac{x dx}{(1+x^2)^2}$. Then $du = \frac{dx}{1+x^2}$,

$$v = \frac{1}{2} \int \frac{2x dx}{(1+x^2)^2} = \frac{-1/2}{1+x^2}, \text{ and}$$

$$\int \frac{x \arctan x}{(1+x^2)^2} dx = -\frac{1}{2} \frac{\arctan x}{1+x^2} + \frac{1}{2} \int \frac{dx}{(1+x^2)^2} \quad \left[\begin{array}{l} x = \tan \theta, \\ dx = \sec^2 \theta d\theta \end{array} \right] \\ = -\frac{1}{2} \frac{\arctan x}{1+x^2} + \frac{1}{2} \int \frac{\sec^2 \theta d\theta}{(\sec^2 \theta)^2} \\ = -\frac{1}{2} \frac{\arctan x}{1+x^2} + \frac{1}{2} \int \cos^2 \theta d\theta \\ = -\frac{1}{2} \frac{\arctan x}{1+x^2} + \frac{\theta}{4} + \frac{\sin \theta \cos \theta}{4} + C \\ = -\frac{1}{2} \frac{\arctan x}{1+x^2} + \frac{1}{4} \arctan x + \frac{1}{4} \frac{x}{1+x^2} + C$$



It follows that

$$\int_0^{\infty} \frac{x \arctan x}{(1+x^2)^2} dx = \lim_{t \rightarrow \infty} \left[-\frac{1}{2} \frac{\arctan x}{1+x^2} + \frac{1}{4} \arctan x + \frac{1}{4} \frac{x}{1+x^2} \right]_0^t \\ = \lim_{t \rightarrow \infty} \left(-\frac{1}{2} \frac{\arctan t}{1+t^2} + \frac{1}{4} \arctan t + \frac{1}{4} \frac{t}{1+t^2} \right) = 0 + \frac{1}{4} \cdot \frac{\pi}{2} + 0 = \frac{\pi}{8}.$$

Convergent.

27. There is an infinite discontinuity at the left endpoint of $[0, 3]$.

$$\int_0^3 \frac{dx}{\sqrt{x}} = \lim_{t \rightarrow 0^+} \int_t^3 \frac{dx}{\sqrt{x}} = \lim_{t \rightarrow 0^+} [2\sqrt{x}]_t^3 = \lim_{t \rightarrow 0^+} (2\sqrt{3} - 2\sqrt{t}) = 2\sqrt{3}. \text{ Convergent}$$

28. There is an infinite discontinuity at the left endpoint of $[0, 3]$.

$$\int_0^3 \frac{dx}{x\sqrt{x}} = \lim_{t \rightarrow 0^+} \int_t^3 \frac{dx}{x^{3/2}} = \lim_{t \rightarrow 0^+} \left[\frac{-2}{\sqrt{x}} \right]_t^3 = \frac{-2}{\sqrt{3}} + \lim_{t \rightarrow 0^+} \frac{2}{\sqrt{t}} = \infty. \text{ Divergent}$$

29. There is an infinite discontinuity at the right endpoint of $[-1, 0]$.

$$\int_{-1}^0 \frac{dx}{x^2} = \lim_{t \rightarrow 0^-} \int_{-1}^t \frac{dx}{x^2} = \lim_{t \rightarrow 0^-} \left[\frac{-1}{x} \right]_{-1}^t = \lim_{t \rightarrow 0^-} \left[-\frac{1}{t} + \frac{1}{-1} \right] = \infty. \text{ Divergent}$$

$$30. \int_1^9 \frac{dx}{\sqrt[3]{x-9}} = \lim_{t \rightarrow 9^-} \int_1^t \frac{dx}{\sqrt[3]{x-9}} = \lim_{t \rightarrow 9^-} \left[\frac{3}{2}(x-9)^{2/3} \right]_1^t = \lim_{t \rightarrow 9^-} \left[\frac{3}{2}(t-9)^{2/3} - \frac{3}{2}(4) \right] = 0 - 6 = -6.$$

Convergent

$$31. \int_{-2}^3 \frac{dx}{x^4} = \int_{-2}^0 \frac{dx}{x^4} + \int_0^3 \frac{dx}{x^4}, \text{ but } \int_{-2}^0 \frac{dx}{x^4} = \lim_{t \rightarrow 0^-} \left[-\frac{x^{-3}}{3} \right]_{-2}^t = \lim_{t \rightarrow 0^-} \left[-\frac{1}{3t^3} - \frac{1}{24} \right] = \infty. \text{ Divergent}$$

$$32. \int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{t \rightarrow 1^-} \int_0^t \frac{dx}{\sqrt{1-x^2}} = \lim_{t \rightarrow 1^-} [\sin^{-1} x]_0^t = \lim_{t \rightarrow 1^-} \sin^{-1} t = \frac{\pi}{2}. \text{ Convergent}$$

33. There is an infinite discontinuity at $x = 1$. $\int_0^{33} (x-1)^{-1/5} dx = \int_0^1 (x-1)^{-1/5} dx + \int_1^{33} (x-1)^{-1/5} dx$.
 $\int_0^1 (x-1)^{-1/5} dx = \lim_{t \rightarrow 1^-} \int_0^t (x-1)^{-1/5} dx = \lim_{t \rightarrow 1^-} \left[\frac{5}{4} (x-1)^{4/5} \right]_0^t = \lim_{t \rightarrow 1^-} \left[\frac{5}{4} (t-1)^{4/5} - \frac{5}{4} \right] = -\frac{5}{4}$ and
 $\int_1^{33} (x-1)^{-1/5} dx = \lim_{t \rightarrow 1^+} \int_t^{33} (x-1)^{-1/5} dx = \lim_{t \rightarrow 1^+} \left[\frac{5}{4} (x-1)^{4/5} \right]_t^{33} = \lim_{t \rightarrow 1^+} \left[\frac{5}{4} \cdot 16 - \frac{5}{4} (t-1)^{4/5} \right] = 20$.
 Thus, $\int_0^{33} (x-1)^{-1/5} dx = -\frac{5}{4} + 20 = \frac{75}{4}$. Convergent

34. $f(y) = 1/(4y - 1)$ has an infinite discontinuity at $y = \frac{1}{4}$.

$$\begin{aligned} \int_{1/4}^1 \frac{1}{4y-1} dy &= \lim_{t \rightarrow (1/4)^+} \int_t^1 \frac{1}{4y-1} dy = \lim_{t \rightarrow (1/4)^+} \left[\frac{1}{4} \ln |4y-1| \right]_t^1 \\ &= \lim_{t \rightarrow (1/4)^+} \left[\frac{1}{4} \ln 3 - \frac{1}{4} \ln(4t-1) \right] = \infty \end{aligned}$$

so $\int_{1/4}^1 \frac{1}{4y-1} dy$ diverges, and hence, $\int_0^1 \frac{1}{4y-1} dy$ diverges. Divergent

35. $\int_0^\pi \sec x dx = \int_0^{\pi/2} \sec x dx + \int_{\pi/2}^\pi \sec x dx$. $\int_0^{\pi/2} \sec x dx = \lim_{t \rightarrow \pi/2^-} \int_0^t \sec x dx$
 $= \lim_{t \rightarrow \pi/2^-} \left[\ln |\sec x + \tan x| \right]_0^t = \lim_{t \rightarrow \pi/2^-} \ln |\sec t + \tan t| = \infty$. Divergent

36. $\int_0^4 \frac{dx}{x^2+x-6} = \int_0^4 \frac{dx}{(x+3)(x-2)} = \int_0^2 \frac{dx}{(x-2)(x+3)} + \int_2^4 \frac{dx}{(x-2)(x+3)}$, and
 $\int_0^2 \frac{dx}{(x-2)(x+3)} = \lim_{t \rightarrow 2^-} \int_0^t \left[\frac{1/5}{x-2} - \frac{1/5}{x+3} \right] dx$ [partial fractions] $\lim_{t \rightarrow 2^-} \left[\frac{1}{5} \ln \left| \frac{x-2}{x+3} \right| \right]_0^t$
 $\lim_{t \rightarrow 2^-} \frac{1}{5} \left[\ln \left| \frac{t-2}{t+3} \right| - \ln \frac{2}{3} \right] = -\infty$. Divergent

37. There is an infinite discontinuity at $x = 0$. $\int_{-1}^0 \frac{e^x}{e^x-1} dx = \int_{-1}^0 \frac{e^x}{e^x-1} dx + \int_0^1 \frac{e^x}{e^x-1} dx$.
 $\int_{-1}^0 \frac{e^x}{e^x-1} dx = \lim_{t \rightarrow 0^-} \int_{-1}^t \frac{e^x}{e^x-1} dx = \lim_{t \rightarrow 0^-} \left[\ln |e^x-1| \right]_{-1}^t = \lim_{t \rightarrow 0^-} \left[\ln |e^t-1| - \ln |e^{-1}-1| \right] = -\infty$.
 so $\int_{-1}^0 \frac{e^x}{e^x-1} dx$ is divergent. The integral $\int_0^1 \frac{e^x}{e^x-1} dx$ also diverges since
 $\int_0^1 \frac{e^x}{e^x-1} dx = \lim_{t \rightarrow 0^+} \int_t^1 \frac{e^x}{e^x-1} dx = \lim_{t \rightarrow 0^+} \left[\ln |e^x-1| \right]_t^1 = \lim_{t \rightarrow 0^+} \left[\ln |e-1| - \ln |e^t-1| \right] = \infty$.
 Divergent

38. $\int_0^2 \frac{x-3}{2x-3} dx = \int_0^{3/2} \frac{x-3}{2x-3} dx + \int_{3/2}^2 \frac{x-3}{2x-3} dx$ and
 $\int \frac{x-3}{2x-3} dx = \frac{1}{2} \int \frac{2x-6}{2x-3} dx = \frac{1}{2} \int \left[1 - \frac{3}{2x-3} \right] dx = \frac{1}{2} x - \frac{3}{4} \ln |2x-3| + C$, so
 $\int_0^{3/2} \frac{x-3}{2x-3} dx = \lim_{t \rightarrow 3/2^-} \frac{1}{4} \left[2x-3 \ln |2x-3| \right]_0^t = \infty$. Divergent

$$39. I = \int_0^2 z^2 \ln z \, dz = \lim_{t \rightarrow 0^+} \int_t^2 z^2 \ln z \, dz \stackrel{101}{=} \lim_{t \rightarrow 0^+} \left[\frac{z^3}{3} (3 \ln z - 1) \right]_t^2$$

$$= \lim_{t \rightarrow 0^+} \left[\frac{8}{9} (3 \ln 2 - 1) - \frac{1}{9} t^3 (3 \ln t - 1) \right] = \frac{8}{3} \ln 2 - \frac{8}{9} - \frac{1}{9} \lim_{t \rightarrow 0^+} [t^3 (3 \ln t - 1)] = \frac{8}{3} \ln 2 - \frac{8}{9} - \frac{1}{9} L.$$

$$\text{Now } L = \lim_{t \rightarrow 0^+} [t^3 (3 \ln t - 1)] = \lim_{t \rightarrow 0^+} \frac{3 \ln t - 1}{t^{-3}} \stackrel{H}{=} \lim_{t \rightarrow 0^+} \frac{3/t}{-3/t^4} = \lim_{t \rightarrow 0^+} (-t^3) = 0. \text{ Thus, } L = 0 \text{ and}$$

$$I = \frac{8}{3} \ln 2 - \frac{8}{9}. \quad \text{Convergent}$$

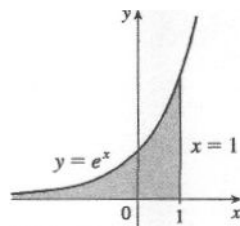
40. Integrate by parts with $u = \ln x$, $dv = dx/\sqrt{x} \Rightarrow du = dx/x$, $v = 2\sqrt{x}$.

$$\int_0^1 \frac{\ln x}{\sqrt{x}} \, dx = \lim_{t \rightarrow 0^+} \int_t^1 \frac{\ln x}{\sqrt{x}} \, dx = \lim_{t \rightarrow 0^+} \left([2\sqrt{x} \ln x]_t^1 - 2 \int_t^1 \frac{dx}{\sqrt{x}} \right) = \lim_{t \rightarrow 0^+} \left(-2\sqrt{t} \ln t - 4[\sqrt{x}]_t^1 \right)$$

$$= \lim_{t \rightarrow 0^+} \left(-2\sqrt{t} \ln t - 4 + 4\sqrt{t} \right) = -4$$

$$\text{since } \lim_{t \rightarrow 0^+} \sqrt{t} \ln t = \lim_{t \rightarrow 0^+} \frac{\ln t}{t^{-1/2}} \stackrel{H}{=} \lim_{t \rightarrow 0^+} \frac{1/t}{-t^{-3/2}/2} = \lim_{t \rightarrow 0^+} (-2\sqrt{t}) = 0. \quad \text{Convergent}$$

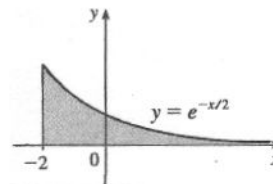
41.



$$\text{Area} = \int_{-\infty}^1 e^x \, dx = \lim_{t \rightarrow -\infty} [e^x]_t^1$$

$$= e - \lim_{t \rightarrow -\infty} e^t = e$$

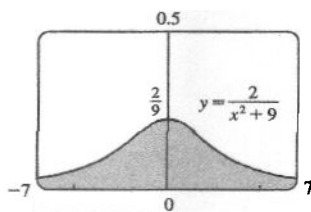
42.



$$\text{Area} = \int_{-2}^{\infty} e^{-x/2} \, dx = -2 \lim_{t \rightarrow \infty} [e^{-x/2}]_{-2}^t$$

$$= -2 \lim_{t \rightarrow \infty} e^{-t/2} + 2e = 2e$$

43.

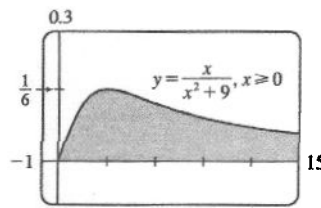


$$\text{Area} = \int_{-\infty}^{\infty} \frac{2}{x^2+9} \, dx = 2 \cdot 2 \int_0^{\infty} \frac{1}{x^2+9} \, dx$$

$$= 4 \lim_{t \rightarrow \infty} \int_0^t \frac{1}{x^2+9} \, dx = 4 \lim_{t \rightarrow \infty} \left[\frac{1}{3} \tan^{-1} \frac{x}{3} \right]_0^t$$

$$= \frac{4}{3} \lim_{t \rightarrow \infty} \left[\tan^{-1} \frac{t}{3} - 0 \right] = \frac{4}{3} \cdot \frac{\pi}{2} = \frac{2\pi}{3}$$

44.



$$\text{Area} = \int_0^{\infty} \frac{x}{x^2+9} \, dx = \lim_{t \rightarrow \infty} \int_0^t \frac{x}{x^2+9} \, dx$$

$$= \lim_{t \rightarrow \infty} \left[\frac{1}{2} \ln(x^2+9) \right]_0^t$$

$$= \frac{1}{2} \lim_{t \rightarrow \infty} [\ln(t^2+9) - \ln 9] = \infty$$

Infinite area