**8.** $f(x)=\sin(x^2)$, $\Delta x=\dfrac{\frac{1}{2}-0}{4}=\dfrac{1}{8}$

(a) $T_4=\frac{1}{8\cdot 2}\left[f(0)+2f\left(\frac{1}{8}\right)+2f\left(\frac{2}{8}\right)+2f\left(\frac{3}{8}\right)+f\left(\frac{1}{2}\right)\right]\approx 0.042743$

(b) $M_4=\frac{1}{8}\left[f\left(\frac{1}{16}\right)+f\left(\frac{3}{16}\right)+f\left(\frac{5}{16}\right)+f\left(\frac{7}{16}\right)\right]\approx 0.040850$

(c) $S_4=\frac{1}{8\cdot 3}\left[f(0)+4f\left(\frac{1}{8}\right)+2f\left(\frac{2}{8}\right)+4f\left(\frac{3}{8}\right)+f\left(\frac{1}{2}\right)\right]\approx 0.041478$

**9.** $f(x)=\dfrac{\ln x}{1+x}$, $\Delta x=\dfrac{2-1}{10}=\dfrac{1}{10}$

(a) $T_{10}=\frac{1}{10\cdot 2}[f(1)+2f(1.1)+2f(1.2)+\cdots+2f(1.8)+2f(1.9)+f(2)]\approx 0.146879$

(b) $M_{10}=\frac{1}{10}[f(1.05)+f(1.15)+\cdots+f(1.85)+f(1.95)]\approx 0.147391$

(c) $S_{10}=\frac{1}{10\cdot 3}[f(1)+4f(1.1)+2f(1.2)+4f(1.3)+2f(1.4)+4f(1.5)+2f(1.6)+4f(1.7)$
$\quad +2f(1.8)+4f(1.9)+f(2)]$
$\approx 0.147219$

**10.** $f(t)=\dfrac{1}{1+t^2+t^4}$, $\Delta t=\dfrac{3-0}{6}=\dfrac{1}{2}$

(a) $T_6=\frac{1}{2\cdot 2}\left[f(0)+2f\left(\frac{1}{2}\right)+2f(1)+2f\left(\frac{3}{2}\right)+2f(2)+2f\left(\frac{5}{2}\right)+f(3)\right]\approx 0.895122$

(b) $M_6=\frac{1}{2}\left[f\left(\frac{1}{4}\right)+f\left(\frac{3}{4}\right)+f\left(\frac{5}{4}\right)+f\left(\frac{7}{4}\right)+f\left(\frac{9}{4}\right)+f\left(\frac{11}{4}\right)\right]\approx 0.895478$

(c) $S_6=\frac{1}{2\cdot 3}\left[f(0)+4f\left(\frac{1}{2}\right)+2f(1)+4f\left(\frac{3}{2}\right)+2f(2)+4f\left(\frac{5}{2}\right)+f(3)\right]\approx 0.898014$

**11.** $f(t)=\sin(e^{t/2})$, $\Delta t=\dfrac{\frac{1}{2}-0}{8}=\dfrac{1}{16}$

(a) $T_8=\frac{1}{16\cdot 2}\left[f(0)+2f\left(\frac{1}{16}\right)+2f\left(\frac{2}{16}\right)+\cdots+2f\left(\frac{7}{16}\right)+f\left(\frac{1}{2}\right)\right]\approx 0.451948$

(b) $M_8=\frac{1}{16}\left[f\left(\frac{1}{32}\right)+f\left(\frac{3}{32}\right)+f\left(\frac{5}{32}\right)+\cdots+f\left(\frac{13}{32}\right)+f\left(\frac{15}{32}\right)\right]\approx 0.451991$

(c) $S_8=\frac{1}{16\cdot 3}\left[f(0)+4f\left(\frac{1}{16}\right)+2f\left(\frac{2}{16}\right)+\cdots+4f\left(\frac{7}{16}\right)+f\left(\frac{1}{2}\right)\right]\approx 0.451976$

**12.** $f(x)=\sqrt{1+\sqrt{x}}$, $\Delta x=\dfrac{4-0}{8}=\dfrac{1}{2}$

(a) $T_8=\frac{1}{2\cdot 2}\left[f(0)+2f\left(\frac{1}{2}\right)+2f(1)+\cdots+2f(3)+2f\left(\frac{7}{2}\right)+f(4)\right]\approx 6.042985$

(b) $M_8=\frac{1}{2}\left[f\left(\frac{1}{4}\right)+f\left(\frac{3}{4}\right)+\cdots+f\left(\frac{13}{4}\right)+f\left(\frac{15}{4}\right)\right]\approx 6.084778$

(c) $S_8=\frac{1}{2\cdot 3}\left[f(0)+4f\left(\frac{1}{2}\right)+2f(1)+4f\left(\frac{3}{2}\right)+2f(2)+4f\left(\frac{5}{2}\right)+2f(3)+4f\left(\frac{7}{2}\right)+f(4)\right]\approx 6.061678$

**13.** $f(x)=e^{1/x}$, $\Delta x=\dfrac{2-1}{4}=\dfrac{1}{4}$

(a) $T_4=\frac{1}{4\cdot 2}[f(1)+2f(1.25)+2f(1.5)+2f(1.75)+f(2)]\approx 2.031893$

(b) $M_4=\frac{1}{4}[f(1.125)+f(1.375)+f(1.625)+f(1.875)]\approx 2.014207$

(c) $S_4=\frac{1}{4\cdot 3}[f(1)+4f(1.25)+2f(1.5)+4f(1.75)+f(2)]\approx 2.020651$

**14.** $f(x)=\sqrt{x}\sin x$, $\Delta x=\dfrac{4-0}{8}=\dfrac{1}{2}$

(a) $T_8=\frac{1}{2\cdot 2}\left\{f(0)+2\left[f\left(\frac{1}{2}\right)+f(1)+f\left(\frac{3}{2}\right)+f(2)+f\left(\frac{5}{2}\right)+f(3)+f\left(\frac{7}{2}\right)\right]+f(4)\right\}\approx 1.732865$

(b) $M_8=\frac{1}{2}\left[f\left(\frac{1}{4}\right)+f\left(\frac{3}{4}\right)+f\left(\frac{5}{4}\right)+f\left(\frac{7}{4}\right)+\cdots+f\left(\frac{13}{4}\right)+f\left(\frac{15}{4}\right)\right]\approx 1.787427$

(c) $S_8=\frac{1}{2\cdot 3}\left[f(0)+4f\left(\frac{1}{2}\right)+2f(1)+4f\left(\frac{3}{2}\right)+2f(2)+4f\left(\frac{5}{2}\right)+2f(3)+4f\left(\frac{7}{2}\right)+f(4)\right]\approx 1.772142$

**15.** $f(x) = \dfrac{\cos x}{x}$, $\Delta x = \dfrac{5-1}{8} = \dfrac{1}{2}$

(a) $T_8 = \frac{1}{2\cdot 2}\left[f(1) + 2f\left(\frac{3}{2}\right) + 2f(2) + \cdots + 2f(4) + 2f\left(\frac{9}{2}\right) + f(5)\right] \approx -0.495333$

(b) $M_8 = \frac{1}{2}\left[f\left(\frac{5}{4}\right) + f\left(\frac{7}{4}\right) + f\left(\frac{9}{4}\right) + f\left(\frac{11}{4}\right) + f\left(\frac{13}{4}\right) + f\left(\frac{15}{4}\right) + f\left(\frac{17}{4}\right) + f\left(\frac{19}{4}\right)\right] \approx -0.543321$

(c) $S_8 = \frac{1}{2\cdot 3}\left[f(1) + 4f\left(\frac{3}{2}\right) + 2f(2) + 4f\left(\frac{5}{2}\right) + 2f(3) + 4f\left(\frac{7}{2}\right) + 2f(4) + 4f\left(\frac{9}{2}\right) + f(5)\right]$
$\approx -0.526123$

**16.** $f(x) = \ln(x^3 + 2)$, $\Delta x = \dfrac{6-4}{10} = \dfrac{1}{5}$

(a) $T_{10} = \frac{1}{5\cdot 2}\left[f(4) + 2f(4.2) + 2f(4.4) + \cdots + 2f(5.6) + 2f(5.8) + f(6)\right] \approx 9.649753$

(b) $M_{10} = \frac{1}{5}[f(4.1) + f(4.3) + \cdots + f(5.7) + f(5.9)] \approx 9.650912$

(c) $S_{10} = \frac{1}{5\cdot 3}[f(4) + 4f(4.2) + 2f(4.4) + 4f(4.6) + 2f(4.8) + 4f(5)$
$+ 2f(5.2) + 4f(5.4) + 2f(5.6) + 4f(5.8) + f(6)]$
$\approx 9.650526$

**17.** $f(y) = \dfrac{1}{1+y^5}$, $\Delta y = \dfrac{3-0}{6} = \dfrac{1}{2}$

(a) $T_6 = \frac{1}{2\cdot 2}\left[f(0) + 2f\left(\frac{1}{2}\right) + 2f\left(\frac{2}{2}\right) + 2f\left(\frac{3}{2}\right) + 2f\left(\frac{4}{2}\right) + 2f\left(\frac{5}{2}\right) + f(3)\right] \approx 1.064275$

(b) $M_6 = \frac{1}{2}\left[f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) + f\left(\frac{5}{4}\right) + f\left(\frac{7}{4}\right) + f\left(\frac{9}{4}\right) + f\left(\frac{11}{4}\right)\right] \approx 1.067416$

(c) $S_6 = \frac{1}{2\cdot 3}\left[f(0) + 4f\left(\frac{1}{2}\right) + 2f\left(\frac{2}{2}\right) + 4f\left(\frac{3}{2}\right) + 2f\left(\frac{4}{2}\right) + 4f\left(\frac{5}{2}\right) + f(3)\right] \approx 1.074915$

**18.** $f(x) = \dfrac{e^x}{x}$, $\Delta x = \dfrac{4-2}{10} = \dfrac{1}{5}$

(a) $T_{10} = \frac{1}{5\cdot 2}\{f(2) + 2[f(2.2) + f(2.4) + f(2.6) + \cdots + f(3.8)] + f(4)\} \approx 14.704592$

(b) $M_{10} = \frac{1}{5}[f(2.1) + f(2.3) + f(2.5) + f(2.7) + \cdots + f(3.7) + f(3.9)] \approx 14.662669$

(c) $S_{10} = \frac{1}{5\cdot 3}[f(2) + 4f(2.2) + 2f(2.4) + 4f(2.6) + \cdots + 2f(3.6) + 4f(3.8) + f(4)] \approx 1$

**19.** $f(x) = e^{-x^2}$, $\Delta x = \dfrac{2-0}{10} = \dfrac{1}{5}$

(a) $T_{10} = \frac{1}{5\cdot 2}\{f(0) + 2[f(0.2) + f(0.4) + \cdots + f(1.8)] + f(2)\} \approx 0.881839$
$M_{10} = \frac{1}{5}[f(0.1) + f(0.3) + f(0.5) + \cdots + f(1.7) + f(1.9)] \approx 0.882202$

(b) $f(x) = e^{-x^2}$, $f'(x) = -2xe^{-x^2}$, $f''(x) = (4x^2 - 2)e^{-x^2}$, $f'''(x) = 4x(3 - 2x^2)e^{-x^2}$.
$f'''(x) = 0 \;\Leftrightarrow\; x = 0$ or $x = \pm\sqrt{\frac{3}{2}}$. So to find the maximum value of $|f''(x)|$ on $[0, 2]$, we consider its values at $x = 0$, $x = 2$, and $x = \sqrt{\frac{3}{2}}$. $|f''(0)| = 2$, $|f''(2)| \approx 0.2564$ and $\left|f''\left(\sqrt{\frac{3}{2}}\right)\right|$
Thus, taking $K = 2$, $a = 0$, $b = 2$, and $n = 10$ in Theorem 3, we get $|E_T| \leq 2\cdot 2^3/(12\cdot 10^2) =$ and $|E_M| \leq |E_T|/2 \leq 0.00\overline{6}$.

(c) Take $K = 2$ [as in part (b)] in Theorem 3. $|E_T| \leq \dfrac{K(b-a)^3}{12n^2} \leq 10^{-5} \;\Leftrightarrow\; \dfrac{2(2-0)^3}{12n^2} \leq 10^{-5}$
$\frac{3}{4}n^2 \geq 10^5 \;\Leftrightarrow\; n \geq 365.1\ldots \;\Leftrightarrow$
Theorem 3 to get $|E_M| \leq 10^{-5} \Leftrightarrow$

**20.** (a) $T_8 = \frac{1}{8\cdot 2}\left\{f(0) + 2\left[f\left(\frac{1}{8}\right) + f\left(\frac{2}{8}\right) + \cdots + f\left(\frac{7}{8}\right)\right] + f(1)\right\} \approx 0.902333$
$M_8 = \frac{1}{8}\left[f\left(\frac{1}{16}\right) + f\left(\frac{3}{16}\right) + f\left(\frac{5}{16}\right) + \cdots + f\left(\frac{15}{16}\right)\right] = 0.905620$