Dr. ZABIDAWI

Selected Problems of
Section 8.5

#5) $\displaystyle\int_{-1}^{1} \frac{e^{\arctan y}}{1+y^2}\,dy = I$

let $u = \tan^{-1} y \Rightarrow du = \frac{1}{1+y^2}\,dy$

$\displaystyle\Rightarrow I = \int_{-\pi/4}^{\pi/4} e^{u}\,du = e^{u}\Big|_{-\pi/4}^{\pi/4} = e^{\pi/4} - e^{-\pi/4}$

#10) $\displaystyle\int \frac{x}{x^4+x^2+1}\,dx = I$

let $u = x^2 \Rightarrow du = 2x\,dx \quad , \quad x\,dx = \frac{1}{2}\,du$

$x^4+x^2+1 = u^2+u+1 = u^2+u+\frac{1}{4}-\frac{1}{4}+1 \quad ; \quad ? = \frac{1}{2}$

$= \left(u+\frac{1}{2}\right)^2 + \frac{3}{4}$

$\displaystyle I = \frac{1}{2}\int \frac{du}{\left(u+\frac{1}{2}\right)^2+3/4}$;

let $V = u + \frac{1}{2} \Rightarrow dV = du$

$\displaystyle I = \frac{1}{2}\int \frac{dV}{V^2+3/4} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left(V\cdot\frac{2}{\sqrt{3}}\right) + C$

$\displaystyle = \frac{1}{\sqrt{3}}\tan^{-1}\left(\left(u+\frac{1}{2}\right)\cdot\frac{2}{\sqrt{3}}\right) + C$

$\displaystyle = \frac{1}{\sqrt{3}}\tan^{-1}\left(\left(x^2+\frac{1}{2}\right)\cdot\frac{2}{\sqrt{3}}\right) + C$

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#15) $\displaystyle\int_0^{1/2} \frac{x\,dx}{\sqrt{1-x^2}} = I$

let $u = 1-x^2 \Longrightarrow du = -2x\,dx$; $x\,dx = -\frac{1}{2}du$

$$I = -\frac{1}{2}\int_1^{3/4} \frac{du}{u^{1/2}} = -\frac{1}{2}\, u^{1/2} \cdot 2 \Big|_1^{3/4}$$

$$= -\left[\frac{\sqrt{3}}{2} - 1\right] = 1 - \frac{\sqrt{3}}{2}$$

#20) $\displaystyle\int e^{\sqrt[3]{x}}\,dx = I$

let $t = \sqrt[3]{x}$

$t^3 = x \Longrightarrow 3t^2 dt = dx$

$I = 3\int e^t t^2 dt = 3\int t^2 e^t dt$; Now Integrate By Parts

let $\begin{array}{c|c} u = t^2 & dv = e^t dt \\ du = 2t\,dt & v = e^t \end{array}$

$$I = 3\left[t^2 e^t - 2\int t e^t\,dt\right]$$

$\begin{array}{c|c} u = t & dv = e^t dt \\ du = dt & v = e^t \end{array}$

$$I = 3\left[t^2 e^t - 2te^t + 2\int e^t dt\right]$$

$= 3\left[t^2 e^t - 2te^t + 2e^t\right] + C$; But $t = \sqrt[3]{x}$, $t^2 = x^{2/3}$

$$I = 3e^t\left[t^2 - 2t + 2\right] + C$$

$$= 3e^{\sqrt[3]{x}}\left[x^{2/3} - 2\sqrt[3]{x} + 2\right] + C$$

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#25) $\displaystyle\int \frac{3x^2-2}{x^2-2x-8}\,dx = I$

$$\begin{array}{r|l} & \quad 3 \\ x^2-2x-8 & \quad 3x^2-2 \\ & -3x^2+6x+24 \\ \hline & \quad 6x+22 \end{array}$$

$$I = \int\left(3 + \frac{6x+22}{(x^2-2x-8)}\right)dx = 3x + 2\int \frac{3x+11}{(x-4)(x+2)}\,dx$$

$$\frac{3x+11}{(x-4)(x+2)} = \frac{A}{(x-4)} + \frac{B}{(x+2)}$$

$$A = \left.\frac{3x+11}{(x+2)}\right|_{x=4} = \frac{23}{6} \quad ; \quad \boxed{A = \frac{23}{6}}$$

$$B = \left.\frac{3x+11}{(x-4)}\right|_{x=-2} = \frac{5}{-6} = -\frac{5}{6} \quad ; \quad \boxed{B = \frac{-5}{6}}$$

$$\Rightarrow I = 3x + 2\left[\frac{23}{6}\ln|x-4| - \frac{5}{6}\ln|x+2|\right] + C$$

$$= 3x + \frac{1}{3}\left[23\ln|x-4| - 5\ln|x+2|\right] + C$$

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#30) $\displaystyle\int_{-2}^{2} |x^2-4x|\,dx = I$

when is $x^2-4x<0$??

$x(x-4)<0$

x	$-\infty$	0		4	$+\infty$
x^2-4x	$+$	0	$-$	0	$+$

$\Longrightarrow x^2-4x<0$ for $x \in (0,4)$

$$I = \int_{-2}^{0}(x^2-4x)\,dx - \int_{0}^{2}(x^2-4x)\,dx \qquad \text{because } |x^2-4x| = \begin{cases} x^2-4x & \text{for } x<0 \text{ or } x>4 \\ -(x^2-4x) & \text{for } 0<x<4 \end{cases}$$

Again $|x^2-4| = \begin{cases} x^2-4x & \text{for } x<0 \text{ or } x>4 \\ -(x^2-4x) = 4x-x^2 & \text{for } 0<x<4 \end{cases}$

$$\therefore I = \frac{x^3}{3} - \frac{4x^2}{2}\Big|_{-2}^{0} - \frac{x^3}{3} + 4x\Big|_{0}^{2}$$

$$= 0 - \left(\frac{-8}{3} - 2(-2)^2\right) + \left(-\frac{8}{3} + 8\right) - (0+0)$$

$$= \frac{8}{3} + 8 - \frac{8}{3} + 8 = \underline{\underline{16}}.$$

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#35) $\int_{-1}^{1} x^8 \sin x \, dx$

$f(x) = x^8 \sin x$ = Even function × ODD function

= ODD function.

OR $f(-x) = (-x)^8 \sin(-x)$

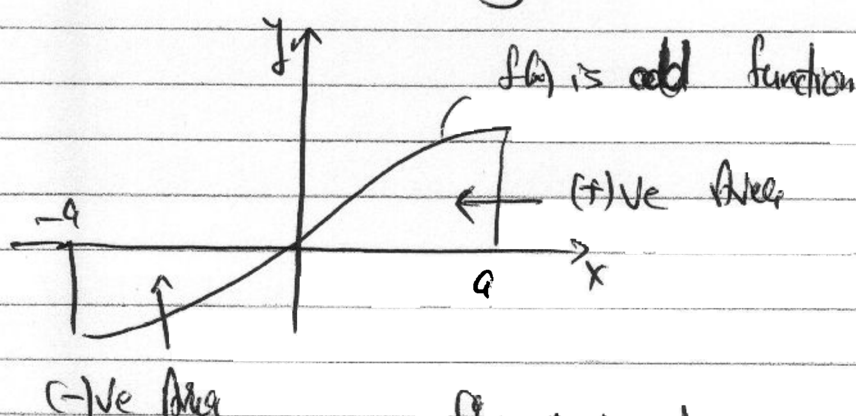
$= x^8(-\sin x) = -x^8 \sin x = -f(x)$

So $f(-x) = -f(x)$

$\Longrightarrow$ that $f(x)$ is an odd function.

When $\int_{-a}^{a} \text{ODD function } dx = 0$

Because odd functions are symmetric wrt origin.



$\Longrightarrow \int_{-a}^{a} \text{odd function } dx = 0.$

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#40) $\int \frac{1}{\sqrt{4y^2-4y-3}}\,dy = I$

$$4y^2-4y-3 = 4(y^2-y)-3 \qquad ? = -\tfrac{1}{2}$$

$$= 4\left(y^2-y+\tfrac{1}{4}-\tfrac{1}{4}\right)-3$$

$$= 4\left(y^2-y+\tfrac{1}{4}\right)-4$$

$$= 4\left(y-\tfrac{1}{2}\right)^2-4 = 4\left[\left(y-\tfrac{1}{2}\right)^2-1\right]$$

$$\Rightarrow I = \frac{1}{2}\int \frac{1}{\sqrt{(y-1/2)^2-1}}\,dy$$

let $u = y-1/2 \Rightarrow du = dy$

$$I = \frac{1}{2}\int \frac{1}{\sqrt{u^2-1}}\,du = \frac{1}{2}$$

Use Identity or formula #20) on page 542

#20) $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x+\sqrt{x^2 \pm a^2}\right|$

$$I = \frac{1}{2}\ln\left|u+\sqrt{u^2-1}\right| + C$$

$$= \frac{1}{2}\ln\left|(y-1/2)+\sqrt{(y-1/2)^2-1}\right| + C$$

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#45) $\int x^5 e^{-x^3}\,dx = \int x^3 \cdot x^2 e^{-x^3}\,dx = I$

let $t = x^3 \Longrightarrow du = 3x^2\,dx \Longrightarrow x^2 dx = \frac{1}{3}\,du$

$I = \frac{1}{3}\int t e^{-t}\,dt$; Now Integrate By Parts

$$\begin{array}{l|l} u = t & dv = e^{-t}\,dt \\ du = dt & v = -e^{-t} \end{array}$$

$$I = \frac{1}{3}\left[-te^{-t} + \int e^{-t}\,dt\right]$$

$$= \frac{1}{3}\left[-te^{-t} - e^{-t}\right] + C$$

$$= -\frac{1}{3}e^{-t}\left[t+1\right] + C$$

$$= -\frac{1}{3}e^{-x^3}\left[x^3+1\right] + C$$