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Solutions to Selected Problems
of Section 8.4

#5) $\dfrac{x^4}{x^4-1}$, first we have to divide

$$\begin{array}{r|l} & 1 \\ \hline x^4-1 & x^4 \\ & -x^4+1 \\ \hline & 1 \end{array}$$

$$\frac{x^4}{x^4-1} = 1+\frac{1}{x^4-1} = 1+\frac{1}{(x^2-1)(x^2+1)} = 1+\frac{1}{(x-1)(x+1)(x^2+1)}$$

$$= 1+\frac{A}{(x-1)}+\frac{B}{(x+1)}+\frac{(Cx+D)}{(x^2+1)}$$

#10) $$\int\frac{1}{(t+4)(t-1)}\,dt = \int\left[\frac{A}{(t+4)}+\frac{B}{(t-1)}\right]dt$$

$$A = \left.\frac{1}{(t-1)}\right|_{t=-4} = -\frac{1}{5}$$

$$B = \left.\frac{1}{t+4}\right|_{t=1} = \frac{1}{5}$$

$$\therefore \int\frac{1}{(t+4)(t-1)}\,dt = -\frac{1}{5}\ln|t+4| + \frac{1}{5}\ln|t-1| + C$$

$$= \frac{1}{5}\ln\left|\frac{t-1}{t+4}\right| + C$$

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#15) $\int_0^1 \frac{2x+3}{(x+1)^2}\,dx$

$$\frac{2x+3}{(x+1)^2} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2}$$

$$B = (2x+3)\Big|_{x=-1} = 1 \quad , \quad \boxed{B=1}$$

$$\frac{2x+3}{(x+1)^2} = \frac{A(x+1)+B}{(x+1)^2} = \frac{Ax+A+B}{(x+1)^2}$$

$$\Rightarrow A=2 \quad , \quad A+B=3$$

$$2+1=3 \quad \checkmark$$

$$\int_0^1 \frac{(2x+3)}{(x+1)^2}\,dx = \int_0^1 \frac{2}{(x+1)}\,dx + \int_0^1 \frac{1}{(x+1)^2}\,dx$$

$$= 2\ln|x+1|\Big|_0^1 - \frac{1}{(x+1)}\Big|_0^1$$

$$= 2\left[\ln(2) - \cancel{\ln 1}^{\,0}\right] - \left[\frac{1}{2} - \frac{1}{1}\right]$$

$$= 2\ln 2 + \frac{1}{2}.$$

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#22) $\displaystyle\int \frac{x^2}{(x-3)(x+2)^2}\,dx$

$$\frac{x^2}{(x-3)(x+2)^2} = \frac{A}{(x-3)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

$$A = \left.\frac{x^2}{(x+2)^2}\right|_{x=3} = \frac{9}{25}, \quad \boxed{A = \frac{9}{25}}$$

$$C = \left.\frac{x^2}{(x-3)}\right|_{x=-2} = \frac{4}{-5} = -\frac{4}{5}$$

$$\frac{x^2}{(x-3)(x+2)^2} = \frac{A(x+2)^2 + B(x+2)(x-3) + C(x-3)}{(x-3)(x+2)^2}$$

$$\Rightarrow x^2 = \frac{A(x^2+4x+4) + B(x^2-x-6) + Cx - 3C}{(x-3)(x+2)^2}$$

$$x^2 = (A+B)x^2 + (4A-B+C)x + 4A-6B-3C$$

$$A+B=1 \Longrightarrow B = 1-A = 1-\frac{9}{25} = \frac{16}{25}, \quad \boxed{B = \frac{16}{25}}$$

$$4A-B+C=0 \quad \text{—— (1)}$$
$$4A-6B-3C=0 \quad \text{—— (2)}$$

$$(1) \Longrightarrow C = B-4A = \frac{16}{25} - \frac{36}{25} = \frac{-20}{25}, \quad \boxed{C = \frac{-20}{25}}$$

$$\int \frac{x^2}{(x-3)(x+2)^2}\,dx = \frac{9}{25}\int\frac{dx}{(x-3)} + \frac{16}{25}\int\frac{dx}{(x+2)} - \frac{4}{5}\int\frac{dx}{(x+2)^2} \qquad \boxed{C = -\frac{4}{5}}$$

$$= \frac{9}{25}\ln|x-3| + \frac{16}{25}\ln|x+2| + \frac{4}{5}\cdot\frac{1}{(x+2)} + C$$

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#25) $\displaystyle\int \frac{10\,dx}{(x-1)(x^2+9)}$

$$\frac{10}{(x-1)(x^2+9)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2+9)}$$

$$A = \left.\frac{10}{(x^2+9)}\right|_{x=1} = \frac{10}{10} = 1, \quad \boxed{A=1}$$

$$\frac{10}{(x-1)(x^2+9)} = \frac{A(x^2+9) + (Bx+C)(x-1)}{(x-1)(x^2+9)}$$

$$10 = Ax^2+9A + Bx^2 - Bx + Cx - C$$
$$= (A+B)x^2 + (C-B)x + 9A - C$$

$$A+B=0 \implies B=-A=-1, \quad \boxed{B=-1}$$
$$C-B=0 \implies C=B=-1, \quad \boxed{C=-1}$$

$$9A-C=0 \quad \checkmark$$

$$\int \frac{10\,dx}{(x-1)(x^2+9)} = \int \frac{dx}{(x-1)} + \int \frac{(-x-1)\,dx}{(x^2+9)}$$
$$= \ln|x-1| - \int \frac{x\,dx}{(x^2+9)} - \int \frac{dx}{x^2+9}$$
$$= \ln|x-1| - \frac{1}{2}\ln(x^2+9) - \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C$$

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#30) $\displaystyle\int \frac{x^3-2x^2+x+1}{x^4+5x^2+4}\,dx$

$$\frac{x^3-2x^2+x+1}{x^4+5x^2+4} = \frac{x^3-2x^2+x+1}{(x^2+4)(x^2+1)} = \frac{Ax+B}{x^2+4} + \frac{Cx+D}{x^2+1}$$

$$\frac{x^3-2x^2+x+1}{x^4+5x^2+4} = \frac{(Ax+B)(x^2+1) + (Cx+D)(x^2+4)}{(x^2+4)(x^2+1)}$$

$$x^3-2x^2+x+1 = Ax^3+Ax+Bx^2+B+Cx^3+4Cx+Dx^2+4D$$

$$= (A+C)x^3+(B+D)x^2+(A+4C)x+B+4D$$

$$\Rightarrow \quad A+C=1 \text{ --- } (1)$$
$$B+D=-2 \text{ --- } (2)$$
$$A+4C=1 \text{ --- } (3)$$
$$B+4D=1 \text{ --- } (4)$$

$$A+C=1 \text{ --- } (1)$$
$$A+4C=1 \text{ --- } (4)$$

$$(1)-(2) \Rightarrow -3C=0 \Rightarrow \boxed{C=0}$$

$$(1) \Rightarrow \boxed{A=1}$$

$$B+D=-2 \text{ --- } (3)$$
$$B+4D=1 \text{ --- } (4)$$

$$(3)-(4) \Rightarrow -3D=-3 \Rightarrow \boxed{D=1}$$
$$(3) \Rightarrow \boxed{B=-3}$$

$$\therefore \int\left(\frac{x^3-2x^2+x+1}{x^4+5x^2+4}\right)dx = \int\frac{(x-3)}{(x^2+4)}\,dx + \int\frac{dx}{(x^2+1)}$$

$$= \frac{1}{2}\ln|x^2+4| - \frac{3}{2}\tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}x + C$$

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#35) $$\int \frac{dx}{x^4-x^2} = \int \frac{dx}{x^2(x^2-1)} = \int \frac{dx}{x^2(x-1)(x+1)}$$

$$\frac{1}{x^2(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{(x-1)} + \frac{D}{(x+1)}$$

$$B = \left.\frac{1}{(x-1)(x+1)}\right|_{x=0} = -1 \quad , \quad \boxed{B=-1}$$

$$C = \left.\frac{1}{x^2(x+1)}\right|_{x=1} = \frac{1}{2} \quad , \quad \boxed{C = 1/2}$$

$$D = \left.\frac{1}{x^2(x-1)}\right|_{x=-1} = -1/2 \quad , \quad \boxed{D = -1/2}$$

$$\frac{1}{x^2(x-1)(x+1)} = \frac{Ax(x^2-1) + B(x^2-1) + Cx^2(x+1) + Dx^2(x-1)}{x^2(x-1)(x+1)}$$

$$\Rightarrow 1 = Ax^3 - Ax + Bx^2 - B + Cx^3 + Cx^2 + Dx^3 - Dx^2$$

$$1 = (A+C+D)x^3 + (B-D+C)x^2 - (A)x - B$$

$$\Rightarrow A+C+D = 0 \text{ --- } (1)$$
$$C+B-D = 0 \text{ --- } (2)$$
$$-A = 0 \text{ --- } (3)$$
$$-B = 1 \text{ --- } (4)$$

Eq (3) $\Rightarrow \boxed{A=0}$

$$\therefore \int \frac{dx}{x^4-x^2} = -\int \frac{dx}{x^2} + 1/2 \int \frac{dx}{(x-1)} - 1/2 \int \frac{dx}{(x+1)}$$

$$= \frac{1}{x} + \frac{1}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + C$$

$$= 1/x + \frac{1}{2} \ln\left|\frac{x-1}{x+1}\right| + C$$

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#40) $\int \frac{dx}{x - \sqrt{x+2}}$

let $u = \sqrt{x+2} \Rightarrow u^2 = x+2$, $x = u^2 - 2$

$2u\,du = dx$

$$\int \frac{dx}{x-\sqrt{x+2}} = 2\int \frac{u\,du}{u^2-2-u} = 2\int \frac{u\,du}{u^2-u-2}$$

$$\frac{u}{u^2-u-2} = \frac{u}{(u-2)(u+1)} = \frac{A}{u-2} + \frac{B}{u+1}$$

$$A = \left.\frac{u}{u+1}\right|_{u=2} = \frac{2}{3} \quad ; \quad \boxed{A = \frac{2}{3}}$$

$$B = \left.\frac{u}{u-2}\right|_{u=-1} = \frac{1}{3} \quad ; \quad \boxed{B = 1/3}$$

$$\int \frac{dx}{x-\sqrt{x+2}} = 2\left[\frac{2}{3}\int \frac{du}{u-2} + \frac{1}{3}\int \frac{du}{u+1}\right]$$

$$= 2\left[\frac{2}{3}\ln|u-2| + \frac{1}{3}\ln|u+1|\right] + C$$

$$= 2\left[\frac{2}{3}\ln\left|\sqrt{x+2}-2\right| + \frac{1}{3}\ln\left|\sqrt{x+2}+1\right|\right] + C$$

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#45) $\displaystyle\int \frac{dx}{\sqrt{x} - \sqrt[3]{x}}$ [Hint: Substitute $u = \sqrt[6]{x}$]

let $u = \sqrt[6]{x}$

$\Rightarrow$ $u^6 = x$, $u^3 = \sqrt{x}$, $u^2 = \sqrt[3]{x}$

$6u^5\,du = dx$

$$\Rightarrow \int \frac{dx}{\sqrt{x} - \sqrt[3]{x}} = 6\int \frac{u^5\,du}{u^3 - u^2} = 6\int \frac{u^5\,du}{u^2(u-1)}$$

$$= 6\int \frac{u^3\,du}{(u-1)}$$

$$\begin{array}{r|l} & u^2 + u + 1 \\ \hline u-1 & u^3 \\ & -u^3 + u^2 \\ \hline & u^2 \\ & -u^2 + u \\ \hline & u \\ & -u + 1 \\ \hline & 1 \end{array}$$

$$\frac{u^3}{u-1} = (u^2+u+1) + \frac{1}{u-1}$$

$$\therefore \int \frac{dx}{\sqrt{x} - \sqrt[6]{x}} = 6\left[\int \left(u^2+u+1 + \frac{1}{u-1}\right) du\right]$$

$$= 6\left[u^3 + \frac{u^2}{2} + u + \ln|u-1|\right] + C$$

$$= 6\left[\frac{\sqrt{x}}{3} + \frac{\sqrt[3]{x}}{2} + \sqrt[6]{x} + \ln\left|\sqrt[6]{x} - 1\right|\right] + C$$

$$= 2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln\left|\sqrt[6]{x} - 1\right| + C$$