38. Use the Limit Comparison Test with $a_n = \sqrt[n]{2} - 1$ and $b_n = 1/n$. Then $\lim\limits_{n\to\infty} \dfrac{a_n}{b_n} = \lim\limits_{n\to\infty} \dfrac{2^{1/n} - 1}{1/n}$

$= \lim\limits_{x\to\infty} \dfrac{2^{1/x} - 1}{1/x} \overset{\text{H}}{=} \lim\limits_{x\to\infty} \dfrac{2^{1/x} \cdot \ln 2 \cdot (-1/x^2)}{-1/x^2} = \lim\limits_{x\to\infty} (2^{1/x} \cdot \ln 2) = 1 \cdot \ln 2 = \ln 2 > 0$. So since $\sum_{n=1}^{\infty} b_n$ diverges (harmonic series), so does $\sum_{n=1}^{\infty} \left(\sqrt[n]{2} - 1\right)$.

Alternate Solution:

$\sqrt[n]{2} - 1 = \dfrac{1}{2^{(n-1)/n} + 2^{(n-2)/n} + 2^{(n-3)/n} + \cdots + 2^{1/n} + 1}$ [rationalize the numerator] $\geq \dfrac{1}{2n}$,

and since $\sum\limits_{n=1}^{\infty} \dfrac{1}{2n} = \dfrac{1}{2} \sum\limits_{n=1}^{\infty} \dfrac{1}{n}$ diverges (harmonic series), so does $\sum\limits_{n=1}^{\infty} \left(\sqrt[n]{2} - 1\right)$ by the Comparison Test.

12.8 Power Series

1. A power series is a series of the form $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$, where x is a variable and the c_n's are constants called the coefficients of the series.

More generally, a series of the form $\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \cdots$ is called a power series in $(x-a)$ or a power series centered at a or a power series about a, where a is a constant.

2. (a) Given the power series $\sum_{n=0}^{\infty} c_n (x-a)^n$, the radius of convergence is:

(i) 0 if the series converges only when $x = a$

(ii) ∞ if the series converges for all x, or

(iii) a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$.

In most cases, R can be found by using the Ratio Test.

(b) The interval of convergence of a power series is the interval that consists of all values of x for which the series converges. Corresponding to the cases in part (a), the interval of convergence is: (i) the single point $\{a\}$, (ii) all real numbers; that is, the real number line $(-\infty, \infty)$, or (iii) an interval with endpoints $a - R$ and $a + R$ which can contain neither, either, or both of the endpoints. In this case, we must test the series for convergence at each endpoint to determine the interval of convergence.

3. If $a_n = \dfrac{x^n}{\sqrt{n}}$, then $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \left|\dfrac{x^{n+1}}{\sqrt{n+1}} \cdot \dfrac{\sqrt{n}}{x}\right| = \lim\limits_{n\to\infty} \left|\dfrac{x}{\sqrt{n+1}/\sqrt{n}}\right| = \lim\limits_{n\to\infty} \dfrac{|x|}{\sqrt{1+1/n}} = |x|$.

By the Ratio Test, the series $\sum\limits_{n=1}^{\infty} \dfrac{x^n}{\sqrt{n}}$ converges when $|x| < 1$, so the radius of convergence $R = 1$. Now we'll check the endpoints, that is, $x = \pm 1$. When $x = 1$, the series $\sum\limits_{n=1}^{\infty} \dfrac{1}{\sqrt{n}}$ diverges because it is a p-series with $p = \frac{1}{2} \leq 1$. When $x = -1$, the series $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{\sqrt{n}}$ converges by the Alternating Series Test. Thus, the interval of convergence is $I = [-1, 1)$.

4. If $a_n = \dfrac{(-1)^n x^n}{n+1}$, then $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\left|\dfrac{x^{n+1}}{n+2}\cdot\dfrac{n+1}{x^n}\right| = \lim\limits_{n\to\infty}\dfrac{|x|}{1+1/(n+1)} = |x|$. By the Ratio Test, the series $\sum\limits_{n=0}^{\infty}\dfrac{(-1)^n x^n}{n+1}$ converges when $|x|<1$, so $R=1$. When $x=-1$, the series diverges because it is the harmonic series; when $x=1$, it is the alternating harmonic series, which converges by the Alternating Series Test. Thus, $I=(-1,1]$.

5. If $a_n = \dfrac{(-1)^{n-1}x^n}{n^3}$, then $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\left|\dfrac{(-1)^n x^{n+1}}{(n+1)^3}\cdot\dfrac{n^3}{(-1)^{n-1}x^n}\right| = \lim\limits_{n\to\infty}\left|\dfrac{(-1)xn^3}{(n+1)^3}\right|$

$= \lim\limits_{n\to\infty}\left[\left(\dfrac{n}{n+1}\right)^3|x|\right] = 1^3\cdot|x| = |x|$. By the Ratio Test, the series $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}x^n}{n^3}$ converges when $|x|<1$, so the radius of convergence $R=1$. Now we'll check the endpoints, that is, $x=\pm 1$. When $x=1$, the series $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}}{n^3}$ converges by the Alternating Series Test. When $x=-1$, the series

$\sum\limits_{n=1}^{\infty}\dfrac{(-1)^{n-1}(-1)^n}{n^3} = -\sum\limits_{n=1}^{\infty}\dfrac{1}{n^3}$ converges because it is a constant multiple of a convergent p-series ($p=3>1$).

Thus, the interval of convergence is $I=[-1,1]$.

6. $a_n=\sqrt{n}\,x^n$, so we need $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\dfrac{\sqrt{n+1}\,|x|^{n+1}}{\sqrt{n}\,|x|^n} = \lim\limits_{n\to\infty}\sqrt{1+\dfrac{1}{n}}\,|x| = |x| < 1$ for convergence (by the Ratio Test), so $R=1$. When $x=\pm1$, $\lim\limits_{n\to\infty}|a_n| = \lim\limits_{n\to\infty}\sqrt{n}=\infty$, so the series diverges by the Test for Divergence. Thus, $I=(-1,1)$.

7. If $a_n=\dfrac{x^n}{n!}$, then $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\left|\dfrac{x^{n+1}}{(n+1)!}\cdot\dfrac{n!}{x^n}\right| = \lim\limits_{n\to\infty}\left|\dfrac{x}{n+1}\right| = |x|\lim\limits_{n\to\infty}\dfrac{1}{n+1} = |x|\cdot 0 = 0 < 1$ for *all* real x. So, by the Ratio Test, $R=\infty$, and $I=(-\infty,\infty)$.

8. Here the Root Test is easier. If $a_n=n^n x^n$ then $\lim\limits_{n\to\infty}\sqrt[n]{|a_n|} = \lim\limits_{n\to\infty} n\,|x| = \infty$ if $x\neq 0$, so $R=0$ and $I=\{0\}$.

9. $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\dfrac{(n+1)\,4^{n+1}\,|x|^{n+1}}{n4^n\,|x|^n} = \lim\limits_{n\to\infty}\left(1+\dfrac{1}{n}\right)4\,|x| = 4\,|x|$. Now $4\,|x|<1 \;\Leftrightarrow\; |x|<\frac14$, so by the Ratio Test, $R=\frac14$. When $x=\frac14$, we get the divergent series $\sum_{n=1}^{\infty}(-1)^n n$, and when $x=-\frac14$, we get the divergent series $\sum_{n=1}^{\infty} n$. Thus, $I=\left(-\frac14,\frac14\right)$.

10. If $a_n=\dfrac{x^n}{n3^n}$, then $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\left|\dfrac{x^{n+1}}{(n+1)3^{n+1}}\cdot\dfrac{n3^n}{x^n}\right| = \lim\limits_{n\to\infty}\left|\dfrac{xn}{(n+1)3}\right| = \dfrac{|x|}{3}\lim\limits_{n\to\infty}\dfrac{n}{n+1} = \dfrac{|x|}{3}$.

By the Ratio Test, the series converges when $\dfrac{|x|}{3}<1 \;\Leftrightarrow\; |x|<3$, so $R=3$. When $x=-3$, the series is the alternating harmonic series, which converges by the Alternating Series Test. When $x=3$, it is the harmonic series, which diverges. Thus, $I=[-3,3)$.

11. $a_n=\dfrac{(-2)^n x^n}{\sqrt[4]{n}}$, so $\lim\limits_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty}\dfrac{2^{n+1}\,|x|^{n+1}}{\sqrt[4]{n+1}}\cdot\dfrac{\sqrt[4]{n}}{2^n\,|x|^n} = \lim\limits_{n\to\infty}2\,|x|\sqrt[4]{\dfrac{n}{n+1}} = 2\,|x|$, so by the Ratio Test, the series converges when $2\,|x|<1 \;\Leftrightarrow\; |x|<\frac12$, so $R=\frac12$. When $x=-\frac12$, we get the divergent

p-series $\sum\limits_{n=1}^{\infty} \dfrac{1}{\sqrt[4]{n}}$ $\left(p = \frac{1}{4} \le 1\right)$. When $x = \frac{1}{2}$, we get the series $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{\sqrt[4]{n}}$, which converges by the Alternating Series Test. Thus, $I = \left(-\frac{1}{2}, \frac{1}{2}\right]$.

12. $a_n = \dfrac{x^n}{5^n n^5}$, so $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \left|\dfrac{x^{n+1}}{5^{n+1}(n+1)^5} \cdot \dfrac{5^n n^5}{x^n}\right| = \lim\limits_{n\to\infty} \dfrac{|x|}{5}\left(\dfrac{n}{n+1}\right)^5 = \dfrac{|x|}{5}$. By the Ratio Test, the series converges when $|x|/5 < 1 \iff |x| < 5$, so $R = 5$. When $x = -5$, we get the series $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{n^5}$, which converges by the Alternating Series Test. When $x = 5$, we get the convergent p-series $\sum\limits_{n=1}^{\infty} \dfrac{1}{n^5}$ $(p = 5 > 1)$. Thus, $I = [-5, 5]$.

13. If $a_n = (-1)^n \dfrac{x^n}{4^n \ln n}$, then

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{x^{n+1}}{4^{n+1}\ln(n+1)} \cdot \frac{4^n \ln n}{x^n}\right| = \frac{|x|}{4} \lim_{n\to\infty} \frac{\ln n}{\ln(n+1)} = \frac{|x|}{4} \cdot 1 \text{ (by l'Hospital's Rule)} = \frac{|x|}{4}.$$

By the Ratio Test, the series converges when $\dfrac{|x|}{4} < 1 \iff |x| < 4$, so $R = 4$. When $x = -4$, $\sum\limits_{n=2}^{\infty} (-1)^n \dfrac{x^n}{4^n \ln n} = \sum\limits_{n=2}^{\infty} \dfrac{[(-1)(-4)]^n}{4^n \ln n} = \sum\limits_{n=2}^{\infty} \dfrac{1}{\ln n}$. Since $\ln n < n$ for $n \ge 2$, $\dfrac{1}{\ln n} > \dfrac{1}{n}$ and $\sum\limits_{n=2}^{\infty} \dfrac{1}{n}$ is the divergent harmonic series (without the $n = 1$ term), $\sum\limits_{n=2}^{\infty} \dfrac{1}{\ln n}$ is divergent by the Comparison Test. When $x = 4$, $\sum\limits_{n=2}^{\infty} (-1)^n \dfrac{x^n}{4^n \ln n} = \sum\limits_{n=2}^{\infty} (-1)^n \dfrac{1}{\ln n}$, which converges by the Alternating Series Test. Thus, $I = (-4, 4]$.

14. $a_n = (-1)^n \dfrac{x^{2n}}{(2n)!}$, so $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \dfrac{|x|^{2n+2}}{(2n+2)!} \cdot \dfrac{(2n)!}{|x|^{2n}} = \lim\limits_{n\to\infty} \dfrac{|x|^2}{(2n+1)(2n+2)} = 0$. Thus, by the Ratio Test, the series converges for *all* real x and we have $R = \infty$ and $I = (-\infty, \infty)$.

15. If $a_n = \sqrt{n}\,(x-1)^n$, then $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \left|\dfrac{\sqrt{n+1}\,|x-1|^{n+1}}{\sqrt{n}\,|x-1|^n}\right| = \lim\limits_{n\to\infty} \sqrt{1 + \dfrac{1}{n}}\,|x-1| = |x-1|$. By the Ratio Test, the series converges when $|x-1| < 1$ [so $R = 1$] $\iff -1 < x - 1 < 1 \iff 0 < x < 2$. When $x = 0$, the series becomes $\sum_{n=0}^{\infty} (-1)^n \sqrt{n}$, which diverges by the Test for Divergence. When $x = 2$, the series becomes $\sum_{n=0}^{\infty} \sqrt{n}$, which also diverges by the Test for Divergence. Thus, $I = (0, 2)$.

16. If $a_n = n^3(x-5)^n$, $\lim\limits_{n\to\infty} \left|\dfrac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \left|\dfrac{(n+1)^3(x-5)^{n+1}}{n^3(x-5)^n}\right| = \lim\limits_{n\to\infty} \left(1 + \dfrac{1}{n}\right)^3 |x-5| = |x-5|$. By the Ratio Test, the series converges when $|x-5| < 1 \iff -1 < x - 5 < 1 \iff 4 < x < 6$. When $x = 4$, the series becomes $\sum_{n=0}^{\infty} (-1)^n n^3$, which diverges by the Test for Divergence. When $x = 6$, the series becomes $\sum_{n=0}^{\infty} n^3$, which also diverges by the Test for Divergence. Thus, $R = 1$ and $I = (4, 6)$.

17. If $a_n = (-1)^n \dfrac{(x+2)^n}{n2^n}$, then

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left[\frac{|x+2|^{n+1}}{(n+1)2^{n+1}}\cdot\frac{n2^n}{|x+2|^n}\right] = \lim_{n\to\infty}\frac{n}{n+1}\cdot\frac{|x+2|}{2} = \frac{|x+2|}{2}.$$ By the Ratio Test, the series converges when $\dfrac{|x+2|}{2} < 1 \;\Leftrightarrow\; |x+2| < 2$ [so $R = 2$] $\Leftrightarrow\; -2 < x+2 < 2 \;\Leftrightarrow\; -4 < x < 0$.

When $x = -4$, the series becomes $\sum\limits_{n=1}^{\infty}(-1)^n\dfrac{(-2)^n}{n2^n} = \sum\limits_{n=1}^{\infty}\dfrac{2^n}{n2^n} = \sum\limits_{n=1}^{\infty}\dfrac{1}{n}$, which is the divergent harmonic series.

When $x = 0$, the series is $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n}{n}$, the alternating harmonic series, which converges by the Alternating Series Test. Thus, $I = (-4, 0]$.

18. If $a_n = \dfrac{(-2)^n}{\sqrt{n}}(x+3)^n$, then

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(-2)^{n+1}(x+3)^{n+1}}{\sqrt{n+1}}\cdot\frac{\sqrt{n}}{(-2)^n(x+3)^n}\right| = \lim_{n\to\infty}\frac{2\,|x+3|}{\sqrt{1+1/n}} = 2\,|x+3| < 1 \;\Leftrightarrow$$

$|x+3| < \frac{1}{2}$ [so $R = \frac{1}{2}$] $\Leftrightarrow\; -\frac{7}{2} < x < -\frac{5}{2}$. When $x = -\frac{7}{2}$, the series becomes $\sum\limits_{n=1}^{\infty}\dfrac{1}{\sqrt{n}}$, which diverges because it is a p-series with $p = \frac{1}{2} \le 1$. When $x = -\frac{5}{2}$, the series becomes $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n}{\sqrt{n}}$, which converges by the Alternating Series Test. Thus, $I = \left(-\frac{7}{2}, -\frac{5}{2}\right]$.

19. If $a_n = \dfrac{(x-2)^n}{n^n}$, then $\lim\limits_{n\to\infty}\sqrt[n]{|a_n|} = \lim\limits_{n\to\infty}\dfrac{|x-2|}{n} = 0$, so the series converges for all x (by the Root Test). $R = \infty$ and $I = (-\infty, \infty)$.

20. $$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(3x-2)^{n+1}}{(n+1)\,3^{n+1}}\cdot\frac{n3^n}{(3x-2)^n}\right| = \lim_{n\to\infty}\left(\frac{|3x-2|}{3}\cdot\frac{1}{1+1/n}\right) = \frac{|3x-2|}{3} = \left|x-\tfrac{2}{3}\right|,$$ so by the Ratio Test, the series converges when $\left|x-\frac{2}{3}\right| < 1 \;\Leftrightarrow\; -\frac{1}{3} < x < \frac{5}{3}$. $R = 1$. When $x = -\frac{1}{3}$, the series is $\sum\limits_{n=1}^{\infty}\dfrac{(-1)^n}{n}$, the convergent alternating harmonic series. When $x = \frac{5}{3}$, the series becomes the divergent harmonic series. Thus, $I = \left[-\frac{1}{3}, \frac{5}{3}\right)$.

21. $a_n = \dfrac{n}{b^n}(x-a)^n$, where $b > 0$.

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{(n+1)\,|x-a|^{n+1}}{b^{n+1}}\cdot\frac{b^n}{n\,|x-a|^n} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)\frac{|x-a|}{b} = \frac{|x-a|}{b}.$$

By the Ratio Test, the series converges when $\dfrac{|x-a|}{b} < 1 \;\Leftrightarrow\; |x-a| < b$ [so $R = b$] $\Leftrightarrow$ $-b < x-a < b \;\Leftrightarrow\; a-b < x < a+b$. When $|x-a| = b$, $\lim\limits_{n\to\infty}|a_n| = \lim\limits_{n\to\infty} n = \infty$, so the series diverges. Thus, $I = (a-b, a+b)$.

22. $a_n = \dfrac{n(x-4)^n}{n^3+1}$, so

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{(n+1)\,|x-4|^{n+1}}{(n+1)^3+1}\cdot\frac{n^3+1}{n\,|x-4|^n} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)\frac{n^3+1}{n^3+3n^2+3n+2}\,|x-4| = |x-4|.$$

By the R

When $|x$

$\sum_{n=1}^{\infty}\frac{1}{n^{\cdot}}$

23. If $a_n = n$

as $n \to \infty$

24. $a_n = \frac{}{2\cdot}$

$\lim_{n\to\infty}\left|\frac{a_{n}}{a_{n}}\right.$

converges

25. $\lim_{n\to\infty}\left|\frac{a_{n+}}{a_n}\right.$

series conv

$R = \frac{1}{4}$. Wl

$x = 0$, the s

26. If $a_n = \frac{(-}{}$

convergence

and when x

27. If $a_n = \frac{x}{(\ln}$

Root Test.

28. If $a_n = \frac{2\cdot}{1\cdot 3}$

$R = 1$. If x

the correspon

$I = (-1, 1)$.

29. (a) We are giv

at least $-$

(b) It does not

convergen

By the Ratio Test, the series converges when $|x-4|<1$ [so $R=1$] $\Leftrightarrow$ $-1<x-4<1$ $\Leftrightarrow$ $3<x<5$.

When $|x-4|=1$, $\displaystyle\sum_{n=1}^{\infty}|a_n|=\sum_{n=1}^{\infty}\frac{n}{n^3+1}$, which converges by comparison with the convergent p-series $\sum_{n=1}^{\infty}\frac{1}{n^2}$ $(p=2>1)$. Thus, $I=[3,5]$.

23. If $a_n=n!(2x-1)^n$, then $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left|\frac{(n+1)!(2x-1)^{n+1}}{n!(2x-1)^n}\right|=\lim_{n\to\infty}(n+1)\,|2x-1|\to\infty$ as $n\to\infty$ for all $x\neq\frac{1}{2}$. Since the series diverges for all $x\neq\frac{1}{2}$, $R=0$ and $I=\left\{\frac{1}{2}\right\}$.

24. $\displaystyle a_n=\frac{n^2x^n}{2\cdot4\cdot6\cdot\cdots\cdot(2n)}=\frac{n^2x^n}{2^n n!}=\frac{nx^n}{2^n(n-1)!}$, so

$\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\frac{(n+1)\,|x|^{n+1}}{2^{n+1}n!}\cdot\frac{2^n(n-1)!}{n\,|x|^n}=\lim_{n\to\infty}\frac{n+1}{n^2}\frac{|x|}{2}=0$. Thus, by the Ratio Test, the series converges for *all* real x and we have $R=\infty$ and $I=(-\infty,\infty)$.

25. $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}\left[\frac{|4x+1|^{n+1}}{(n+1)^2}\cdot\frac{n^2}{|4x+1|^n}\right]=\lim_{n\to\infty}\frac{|4x+1|}{(1+1/n)^2}=|4x+1|$, so by the Ratio Test, the series converges when $|4x+1|<1$ $\Leftrightarrow$ $-1<4x+1<1$ $\Leftrightarrow$ $-2<4x<0$ $\Leftrightarrow$ $-\frac{1}{2}<x<0$, so $R=\frac{1}{4}$. When $x=-\frac{1}{2}$, the series becomes $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}$, which converges by the Alternating Series Test. When $x=0$, the series becomes $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^2}$, a convergent p-series $(p=2>1)$. $I=\left[-\frac{1}{2},0\right]$.

26. If $a_n=\dfrac{(-1)^n(2x+3)^n}{n\ln n}$, then we need $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=|2x+3|\lim_{n\to\infty}\frac{n\ln n}{(n+1)\ln(n+1)}=|2x+3|<1$ for convergence, so $-2<x<-1$ and $R=\frac{1}{2}$. When $x=-2$, $\displaystyle\sum_{n=2}^{\infty}a_n=\sum_{n=2}^{\infty}\frac{1}{n\ln n}$, which diverges (Integral Test), and when $x=-1$, $\displaystyle\sum_{n=2}^{\infty}a_n=\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln n}$, which converges (Alternating Series Test), so $I=(-2,-1]$.

27. If $a_n=\dfrac{x^n}{(\ln n)^n}$, then $\displaystyle\sqrt[n]{|a_n|}=\lim_{n\to\infty}\frac{|x|}{\ln n}=0<1$ for all x, so $R=\infty$ and $I=(-\infty,\infty)$ by the

28. If $a_n=\dfrac{2\cdot4\cdot6\cdot\cdots\cdot(2n)\,x^n}{1\cdot3\cdot5\cdot\cdots\cdot(2n-1)}$, then we need $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=\lim_{n\to\infty}|x|\left(\frac{2n+2}{2n+1}\right)=|x|<1$ for convergence, so $R=1$. If $x=\pm1$, $|a_n|=\dfrac{2\cdot4\cdot6\cdot\cdots\cdot(2n)}{1\cdot3\cdot5\cdot\cdots\cdot(2n-1)}>1$ for all n since each integer in the numerator is larger than