Solutions To Selected Problems of Section 8.3

#21) $\int \sqrt{2x - x^2}\, dx$, $2x - x^2 = -(x^2 - 2x + 1 - 1) = 1 - (x-1)^2$

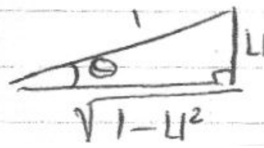
$$\int \sqrt{2x - x^2}\, dx = \int \sqrt{1-(x-1)^2}\, dx$$

let $u = x - 1 \implies du = dx$

$$\int \sqrt{1-(x-1)^2}\, dx = \int \sqrt{1-u^2}\, du$$

let $u = \sin\theta \implies \theta = \sin^{-1} u$, $-\pi/2 \le \theta \le \pi/2$

$du = \cos\theta\, d\theta$



$$\int \sqrt{1-u^2}\, du = \int \sqrt{1-\sin^2\theta}\, \cos\theta\, d\theta$$

$$= \int \cos^2\theta\, d\theta = \int \left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2}\theta + \frac{\sin 2\theta}{4} + C = \frac{1}{2}\theta + \frac{2\sin\theta\cos\theta}{4} + C$$

$$= \frac{1}{2}\sin^{-1}(u) + \frac{1}{2}\, u\, \sqrt{1-u^2} + C$$

$$= \frac{1}{2}\sin^{-1}(x-1) + \frac{1}{2}(x-1)\sqrt{2x-x^2} + C .$$

#20) $\displaystyle\int \frac{dx}{(4x^2-25)^{3/2}} = \int \frac{dx}{8\left(x^2-\left(\frac{5}{2}\right)^2\right)^{3/2}}$

let $x = \frac{5}{2}\sec\theta \rightarrow \sec\theta = \frac{2x}{5} \rightarrow \theta$

$0 \leq \theta < \pi/2$ or $\pi \leq \theta < 3\pi/2$.

$dx = \frac{5}{2}\sec\theta\tan\theta\, d\theta$

$$= \frac{1}{8}\cdot\frac{5}{2}\int \frac{\quad}{\left(\frac{25}{4}\sec^2\theta - \frac{25}{4}\right)^{3/2}} = \frac{5}{16}\cdot\frac{8}{125}\int \frac{\sec\theta\tan\theta\ \theta}{\tan^3\theta}$$

$$\frac{\sec\theta}{\tan^2\theta} = \frac{1}{\cos\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta} = \frac{\cos\theta}{\sin^2\theta}$$

$$= \frac{1}{50} \quad \frac{\cos\theta}{\sin^2\theta}\, d\theta$$

let $u = \sin\theta$ $\quad \cos\theta$

$$\rightarrow \quad \int \frac{du}{u^2} = \frac{1}{50}\left(\frac{u^{-1}}{-1}\right) + C$$

$$= -\frac{1}{50}\cdot\frac{1}{u} + C$$

$$= -\frac{1}{50}\cdot\frac{1}{\sin\theta} + C$$

$$-\frac{1}{50}\cdot\frac{2x}{\sqrt{4x^2-25}} + C$$

$$= \frac{-x}{25\sqrt{4x^2-25}} + C.$$