Solution To Problems of Section 8.2:

#1) $\int_0^{\pi/2} \sin^2 3x\,dx = \int_0^{\pi/2} \left(\frac{1-\cos 6x}{2}\right)dx$

$= \frac{x}{2} - \frac{1}{2}\frac{\sin 6x}{6}\Big|_0^{\pi/2} = \left(\frac{\pi}{4} - \frac{1}{12}\sin 3\pi\right) - (0-0) = \pi/4.$

#2) $\int_0^{\pi/2} \cos^2 x\,dx = \int_0^{\pi/2} \left(\frac{1+\cos 2x}{2}\right)dx = \left(\frac{1}{2}x + \frac{1}{4}\sin 2x\right)\Big|_0^{\pi/2}$

$= \left(\frac{\pi}{4} + 0\right) - (0+0) = \pi/4$

#3) $\int \cos^4 x\,dx = \int (\cos^2 x)^2 dx = \int \left(\frac{1+\cos 2x}{2}\right)^2 dx$

$= \frac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)\,dx$

$= \frac{1}{4}\int \left(1 + 2\cos 2x + \frac{(1+\cos 4x)}{2}\right)dx$

$= \frac{1}{4}\left[x + \sin 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x\right] + C$

#4) $\int \sin^3 x\,dx = \int (1-\cos^2 x)\sin x\,dx$

let $u = \cos x \Rightarrow du = -\sin x\,dx$

$= -\int (1-u^2)\,du = -\left(u - \frac{u^3}{3}\right) + C$

$= -\left(\cos x - \frac{1}{3}\cos^3 x\right) + C$

#5) $\int \sin^3 x \cos^4 x\,dx = \int (1-\cos^2 x)\cos^4 x \sin x\,dx$

let $u = \cos x \Rightarrow du = -\sin x\,dx$

$= -\int (1-u^2)\,u^4\,du = -\int (u^4 - u^6)\,du$

$= -\left(\frac{u^7}{3} - \frac{u^7}{7}\right) + C$

$$\int \sin^3 x \cos^4 x \, dx = -\left[\frac{1}{5} U^5 - \frac{1}{7} U^7\right] + C$$

$$= -\left[\frac{1}{5} \cos^5 x - \frac{1}{7} \cos^7 x\right] + C$$

#6) $$\int \sin^4 x \cos^3 x \, dx = \int \sin^4 x \left(1 - \sin^2 x\right) \cos x \, dx$$

let $u = \sin x \Rightarrow du = \cos x \, dx$

$$= \int u^4 \left(1 - u^2\right) du = \int \left(u^4 - u^6\right) du$$

$$= \frac{1}{5} u^5 - \frac{1}{7} u^7 + c$$

$$= \frac{1}{5} \sin^5 x - \frac{1}{7} \sin^7 x + C.$$

#7) $$\int_0^{\pi/4} \sin^4 x \cos^2 x \, dx = \int_0 \left(\frac{1 - \cos 2x}{2}\right)^2 \left(\frac{1 + \cos 2x}{2}\right) dx$$

$$= \frac{1}{8} \int_0^{\pi/4} \left(1 - 2\cos 2x + \cos^2 2x\right)\left(1 + \cos 2x\right) dx$$

$$= \frac{1}{8} \int_0^{\pi/4} \left(1 + \cos 2x - 2\cos 2x - 2\cos^2 2x + \cos^2 2x + \cos^3 2x\right) dx$$

$$= \frac{1}{8} \int_0^{\pi/4} \left(1 - \cos 2x - \cos^2 2x + \cos^3 2x\right) dx$$

$$= \frac{1}{8} \int_0^{\pi/4} \left(1 - \cos 2x - \left(\frac{1 + \cos 4x}{2}\right)\right) dx + \frac{1}{8} \int_0^{\pi/4} \left(1 - \sin^2 2x\right) \cos 2x \, dx$$

$$= \frac{1}{8}\left[x - \frac{1}{2}\sin 2x - \frac{1}{2}x - \frac{1}{8}\sin 4x\right]\Big|_0^{\pi/4} + \frac{1}{16}\left[\sin 2x - \frac{\sin^3 2x}{3}\right]\Big|_0^{\pi/4}$$

$1\left[\frac{\pi}{} \quad - \frac{\pi}{}\right] + \frac{1}{}\left[1 - 1\right] \quad \frac{1}{}\left[\frac{\pi}{} - 1\right] + \frac{2}{16}$

Solutions To Problems of Section 7.2

8) $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx = \int_0^{\pi/2} \left(\frac{1-\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right) dx$

$= \frac{1}{4}\int_0^{\pi/2} \left(1 - \cos^2 2x\right) dx$

$= \frac{1}{4}\int_0^{\pi/2} \left(1 - \left(\frac{1+\cos 4x}{2}\right)\right) dx$

$= \frac{1}{4}\left[x - \frac{1}{2}x - \frac{1}{8}\sin 4x\right]\Big|_0^{\pi/2}$

$= \frac{1}{4}\left[\pi/2 - \frac{\pi}{4}\right] = \frac{\pi}{16}$

9) $\int (1-\sin 2x)^2 dx = \int \left(1 - 2\sin 2x + \sin^2 2x\right) dx$

$= \int \left(1 - 2\sin 2x + \frac{(1-\cos 4x)}{2}\right) dx$

$= x + \frac{2\cos 2x}{2} + \frac{1}{2}x - \frac{\sin 4x}{8} + C$

10) $\int \sin\left(x+\frac{\pi}{6}\right)\cos x \, dx = \frac{1}{2}\int \left[\sin\left(\frac{\pi}{6}\right) + \sin\left(2x+\frac{\pi}{6}\right)\right] dx$

$= \frac{1}{2}\left[\frac{1}{2}x - \frac{\cos(2x+\pi/6)}{2}\right] + C$

#11) $\int \cos^9 x \sin^5 x \, dx = \int \cos^9 x \sin x (1-\cos^2 x)^2 dx$

let $u = \cos x \rightarrow du = -\sin x \, dx$

$= -\int u^9 (1-u^2)^2 du$

$= -\int u^9 \left[1 - 2u^2 + u^4\right] du$

$$= -\int U^5\left[1-2U^2+U^4\right]dU$$

$$= -\int \left[U^5-2U^7+U^9\right]dU$$

$$= \frac{1}{6}U^6 - \frac{2}{8}U^8 + \frac{1}{9}U^9 + C$$

$$= \frac{1}{6}\cos^6 x - \frac{2}{8}\cos^8 x + \frac{1}{9}\cos^9 x + C$$

12) $\int \sin^6 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^3 dx = \frac{1}{8}\int (1-\cos 2x)^3 dx$

$$= \frac{1}{8}\int \left(1-3\cos 2x+3\cos^2 2x-\cos^3 2x\right)dx$$

$$= \frac{1}{8}\int \left(1-3\cos 2x+3\left(\frac{1+\cos 4x}{2}\right)\right)dx - \frac{1}{8}\int \cos^3 2x\,dx$$

$$= \frac{1}{8}\left[x-\frac{3}{2}\sin 2x+\frac{3}{2}x+\frac{3}{8}\sin 4x\right] - \frac{1}{8}\int (1-\sin^2 2x)\cos 2x\,dx$$

let $U = \sin 2x \longrightarrow dU = 2\cos 2x\,dx$

$$= \frac{1}{8}\left[x-\frac{3}{2}\sin 2x+\frac{3}{2}x+\frac{3}{8}\sin 4x\right] - \frac{1}{8}\int (1-U^2)\,\frac{dU}{2}$$

$$= \frac{1}{8}\left[x-\frac{3}{2}\sin 2x+\frac{3}{2}x+\frac{3}{8}\sin 4x\right] - \frac{1}{16}\left[\sin 2x - \frac{\sin^3 2x}{3}\right] + C$$