Math 3301 Foundations of Geometry
Study Guide for the Final Exam **Name** KEY

Show work to support your solution, whenever possible. Each problem is worth 2 points.

1. Use inductive reasoning to determine the next 2 elements in the set. Explain your reasoning.

(a) 7, 10, 13, 16, 19, 22

Add 3.

(b) 27, 9, 3, 1, $^1/_3$, $^1/_9$, $\frac{1}{27}$, $\frac{1}{81}$

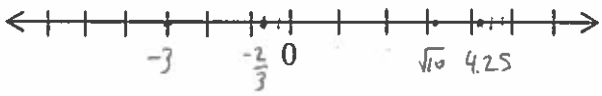
Multiply by $\frac{1}{3}$

2. True or False. Assume Euclidean geometry is true.

(a) Exactly one line can be drawn between two distinct points. True

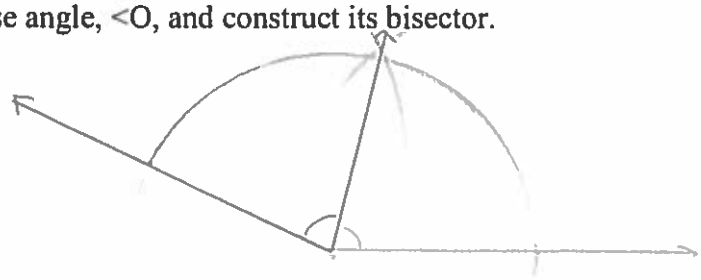
(b) Any three distinct noncollinear points determine a unique plane. True

3. Plot the points associated with 4.25, -3, $^{-2}/_3$, and $\sqrt{10}$ on the number line.

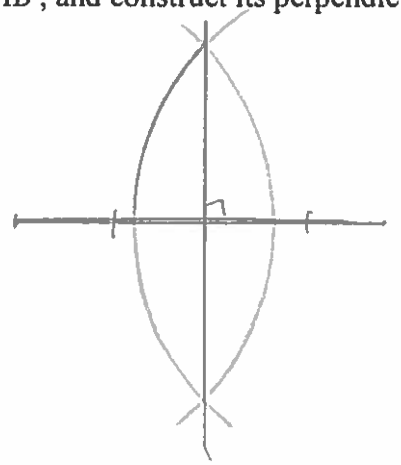


$\sqrt{10} \approx 3.16$

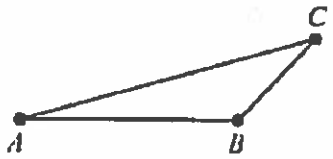
4. Draw an obtuse angle, <O, and construct its bisector.



5. Draw a line segment, $\overline{AB}$, and construct its perpendicular bisector.



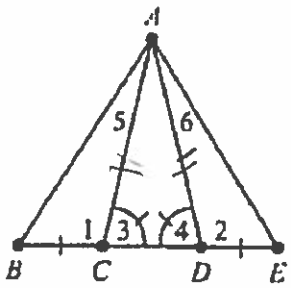
6. Refer to ΔABC below. Assume all angles are unequal, m<B > 90°, and all sides are unequal.



Classify ΔABC using its sides and angles. obtuse scalene triangle

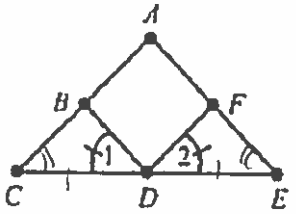
For #7-9, choose <u>any two</u>. Write a two-column or a flowchart proof.

7. *Given:* $\angle 3 \cong \angle 4$
$\overline{BC} \cong \overline{DE}$
Prove: $\angle 5 \cong \angle 6$



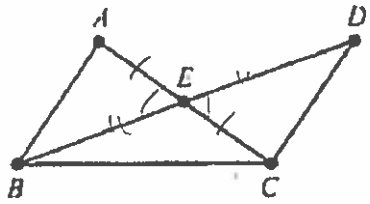
1. $\angle 3 \cong \angle 4$	Given
2. $\overline{AC} \cong \overline{AD}$	Sides opp ≅ ∠s are ≅
3. $\overline{BC} \cong \overline{DE}$	Given
4. ∠1 + ∠3 are suppl., ∠2 + ∠4 are suppl.	Linear pair forms suppl ∠'s.
5. $\angle 1 \cong \angle 2$	Suppl. of ≅ ∠'s are ≅
6. $\Delta ABC \cong \Delta AED$	SAS
7. $\angle 5 \cong \angle 6$	CPCTC

8. *Given:* $\overline{AC} \cong \overline{AE}$
D is the midpoint of $\overline{CE}$
$\angle 1 \cong \angle 2$
Prove: $\overline{BD} \cong \overline{FD}$



1. $\overline{AC} \cong \overline{AE}$	Given
2. $\angle C \cong \angle E$	Angles opp ≅ sides are ≅
3. D is the midpoint of $\overline{CE}$	Given
4. $\overline{CD} \cong \overline{DE}$	Defn. midpoint
5. $\Delta BCD \cong \Delta FED$	ASA
6. $\overline{BD} \cong \overline{FD}$	CPCTC

9. *Given:* E is the midpoint of $\overline{AC}$
E is the midpoint of $\overline{BD}$
Prove: $\overline{AB} \cong \overline{CD}$



1. E is the midpoint of $\overline{AC}$	Given
2. $\overline{AE} \cong \overline{EC}$	Defn midpoint
3. E is the midpoint of $\overline{BD}$	Given
4. $\overline{BE} \cong \overline{ED}$	Defn midpoint
5. $\angle AEB \cong \angle CED$	Vertical ∠s are ≅
6. $\Delta AEB \cong \Delta CED$	SAS
7. $\overline{AB} \cong \overline{CD}$	CPCTC

10. True or False.

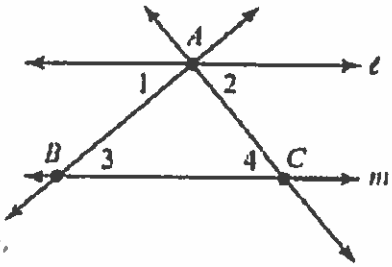
(a) The circumcenter of a triangle is the point of concurrency where the perpendicular bisectors intersect. True

(b) The orthocenter is formed by the intersection of the altitudes of a triangle. True

11. Write a two-column or a flowchart proof.

Given: $\ell \parallel m$ and $\angle 1 \cong \angle 2$

Prove: $\triangle ABC$ is isosceles



1. $\ell \parallel m$ — Given
2. $\angle 2 \cong \angle 4$, $\angle 1 \cong \angle 3$ — $\parallel \rightarrow$ Alt int ∠'s ≅
3. $\angle 1 \cong \angle 2$ — Given
4. $\angle 3 \cong \angle 4$ — Symm/Transitive/Subst.
5. $\overline{AB} \cong \overline{AC}$ — sides opp ≅ ∠'s are ≅
6. $\triangle ABC$ is isosceles — Def. Isosceles △

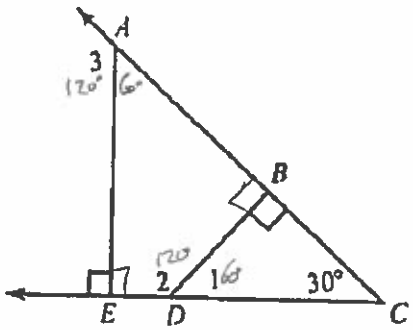
12. True or false. If false, make a correction to make the statement true.

(a) A triangle must have at least two acute angles. True

(b) ~~An~~ A right isosceeles triangle must have a right angle. False

13. *Given:* $\overline{AE} \perp \overline{ED}$, $\overline{DB} \perp \overline{AC}$, $m\angle C = 30°$

Find: the measures of the numbered angles.



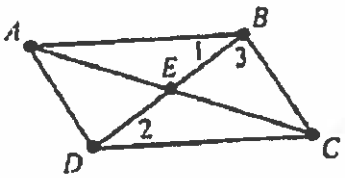
$m\angle 1 = 60°$

$m\angle 2 = m\angle 3 = 120°$

14. What is the measure of each angle in a regular hexagon? Show your reasoning.

$\frac{720°}{6} = 120°$ or $\frac{360}{6} = 60$ $180 - 60 = 120°$

15. Refer to the figure in which ABCD is a parallelogram. True or False.

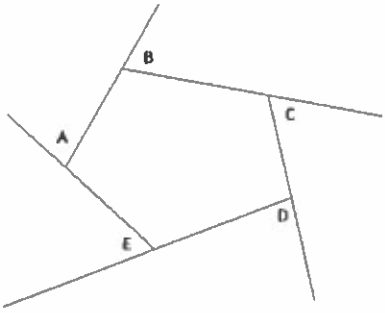


(a) $\overline{AD} \cong \overline{BC}$ True

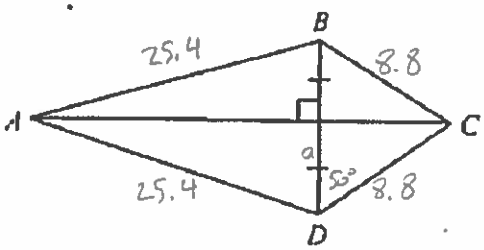
(b) $\angle 1 \cong \angle 2$ True

16. For this convex pentagon, find the <u>sum</u> of the measures of the interior angles.

$(5-2)\cdot 180^\circ = 540^\circ$



17. In kite ABCD, $\overline{AC}$ is the perpendicular bisector of $\overline{BD}$. If $AB = 25.4$ inches, the perimeter of the kite is 68.4 inches, m<BAC = 25°, and m<BDC = 50°, find BD.



$68.4 - 2\cdot 25.4$

$= 68.4 - 50.8 = \frac{17.6}{2} = 8.8$

$\cos 50^\circ = \frac{a}{8.8}$

$a = 8.8\cos 50^\circ \approx 5.7$ in.

$BD = 2(5.7\text{ in}) = 11.4''$

Accept 11.3"

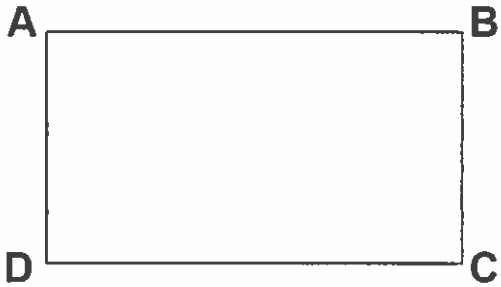
18. True or False. If false, make a correction to make the statement true.

(a) The diagonals of a rhombus bisect its angles. True

(b) A square is a rectangle. True

19. Solve.

Given: Rectangle ABCD with $AD = 4x - 1$; $AB = 2x + 3$; $BC = 3x + 9$
Find the value of x and CD.

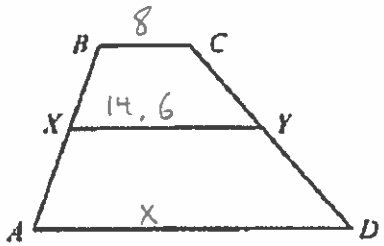


$4x - 1 = 3x + 9$

$x = 10$

$CD = AB = 2(10) + 3 = 23$

20. Given: Trapezoid ABCD, where $\overline{BC} \parallel \overline{AD}$ and X and Y are midpoints of the legs. If XY = 14.6 and BC = 8, find AD.



$\frac{8 + x}{2} = 14.6$

$8 + x = 29.2$

$x = 21.2$

21. True or False. If false, make a correction to make the statement true.

(a) The diagonals of an isosceles trapezoid bisect each other. False

(b) Two pairs of consecutive angles of an isosceles trapezoid are supplementary.

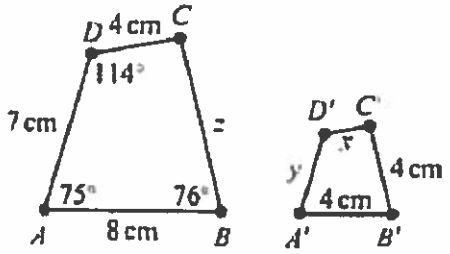
True

22. If ¼ inch on a map represents 50 miles, how many miles are represented by 2 ½ inches?

$\frac{\frac{1}{4}\text{ in}}{50\text{ mi}} = \frac{2\frac{1}{2}''}{x}$ $\quad \frac{1}{4}x = 125$

$x = 500$ miles

23. Refer to the quadrilaterals in the diagram. Assume that ABCD ~ A'B'C'D'.



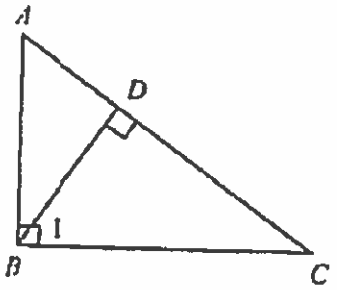
(a) Find the value of y.

3.5 cm

(b) Find m<D'.

114°

Refer to the figure in which $\overline{BD} \perp \overline{AC}$ and $\overline{AB} \perp \overline{BC}$.



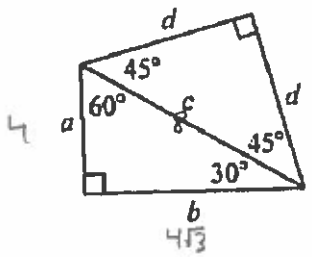
24. Assume BD is an altitude of ΔABC. If AD = 16 yd and DC = 25 yd, find BD to the nearest yard.

BD is a geometric mean b/w AD + DC

$\frac{16}{BD} = \frac{BD}{25}$ $\quad BD^2 = 400$

$BD = \sqrt{400} = 20$ yd

Refer to the figure. Give exact answers (no rounding).



25. If $b = 4\sqrt{3}$ ft, find d.

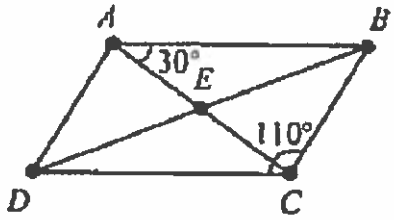
$\frac{8}{\sqrt{2}} = 4\sqrt{2}$

26. A mountain road is inclined 30° with the horizontal. If a pickup truck drives 10 miles on this road, what change in altitude has been achieved?

$\frac{1}{2}(10\text{ mi}) = 5$ miles

$\sin 30° = \frac{v}{10}$ $\quad v = 10 \sin 30° = 5$ mi

27. Refer to the diagram.



(a) Given parallelogram ABCD with the given measures, find m<ABC.

$180 - 110° = 70°$

(b) If $ED = 6.5$ cm, find DB. $2(6.5\text{ cm}) = 13.0$ cm

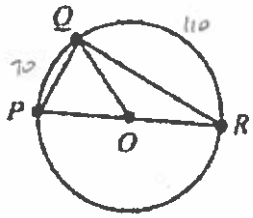
28. Refer to the diagram.

(a) Find m<PQR.

$\frac{1}{2} \cdot 180° = 90°$

(b) Find m<P if m(arc PQ) = 70°.

$\frac{1}{2} \cdot 110 = 55°$



⊙ O, diameter $\overline{PR}$

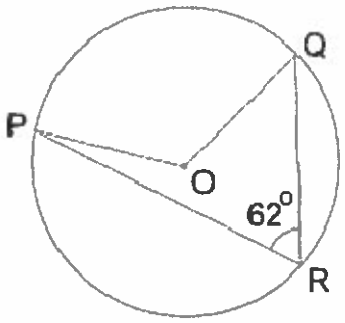
29. Refer to the diagram of circle O. m<PRQ = 62°.

(a) What is the measure of <POQ?

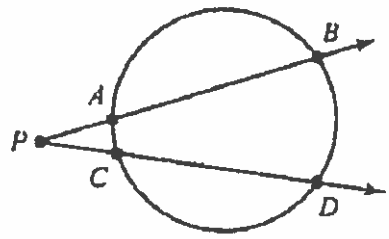
$2 \cdot 62° = 124°$

(b) What is the measure of arc PRQ?

$360° - 124° = 236°$



30. Refer to the diagram.



(a) If m(arc BD) = 75° and m(arc AC) = 27°, find m<P.

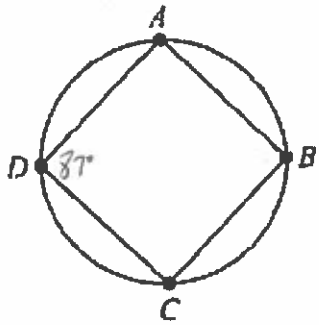
$75 - 27 = 48°$ $\frac{1}{2} \cdot 48 = 24°$

(b) If PB = 18 ft, PA = 4 ft, and PC = 6 ft, find PD.

$18 \cdot 4 = 6 \cdot PD$

$72 = 6 \cdot PD$ $PD = 12$ ft

31. Refer to the diagram.



(a) If m<D = 87°, find m<B.

$180 - 87 = 93$

(b) If $\overline{AD} \parallel \overline{BC}$ and AD = BC, find m<A.

Square → 90°

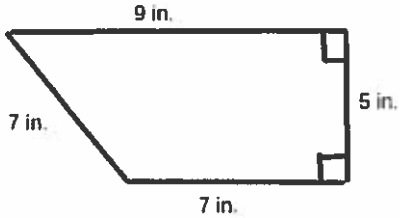
32. Construct a regular hexagon, and inscribe a circle inside it.

Steps: 1 use Ruler for hexagon sidelengths/arcs
2 Draw hexagon
3 ⊥ bisect one side for radius of incircle
4 Draw incircle.

33. What is the measure of each central angle in a 20-gon?

$\frac{360°}{20} = 18°$

34. Find the area and perimeter of the given figure.



A = 40 in² P = 28 in

$\frac{1}{2}(5)(7+9)$ $9+7+7+5$

35. A room has four rectangular walls each measuring 16 ft by 15 ft. Two of the walls have windows measuring 4 ft by 3 ft. If the walls are to be given two coats of paint and 1 gallon of paint covers 400 ft^2, how many gallons will be needed for the job? Round up to the next gallon.

$SA = 4 \cdot 16' \times 15' = 4 \cdot 240\ ft^2 = 960\ ft^2 - 2 \cdot 4' \cdot 3'$

$= 936\ ft^2 \cdot \frac{1\ gal}{400\ ft^2} \approx 2.34\ gal$

3 gallons

36. Find the area of a kite with diagonals measuring 18 in. and 21 in.

$A = \frac{1}{2} \cdot d \cdot d' = \frac{1}{2}(18\ in)(21\ in)$

$= 189\ in^2$

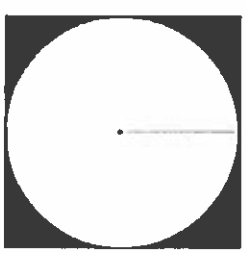
37. A machine part is in the shape of an equilateral triangle 8 inches on a side. A hole with diameter 2 inches is drilled in the center of the part. To the nearest tenth, what is the area of the remaining metal?

$A_\Delta = \frac{8^2\sqrt{3}}{4} = 16\sqrt{3}\ in^2$

$A_O = \pi \cdot 1^2 = \pi\ in^2$

$A_{\Delta - O} = 16\sqrt{3} - \pi\ in^2 \approx 24.57\ in^2$

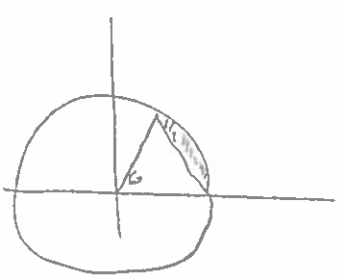
38. Refer to the diagram. If the square has sides which measure 10 inches, find the total area of the four shaded regions. Leave the answer in terms of π.



$10^2 - \pi \cdot 5^2$

$100 - 25\pi\ in^2 \approx 21.5\ in^2$

39. Find the area of a segment of a circle formed by two radii measuring 12 cm that form a 60° central angle. Give the answer rounded to the nearest tenth of a square centimeter.

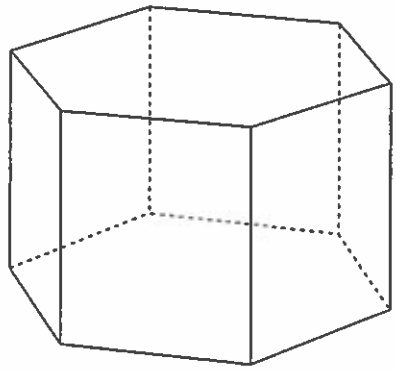


$A_{Segment} = A_{Sector} - A_\Delta$

$= \frac{1}{6}\pi \cdot 12^2 - \frac{12^2\sqrt{3}}{4}$

$24\pi - 36\sqrt{3} \approx 13.0\ cm^2$

40. Determine the number of faces, f, vertices, v, and edges, e, in this polyhedron. Check to see that f + v – e = 2.



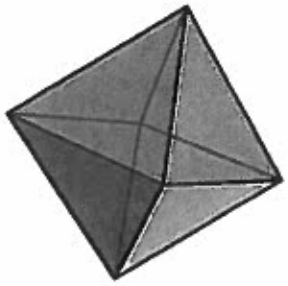
f = 8
v = 12
e = 18

8 + 12 – 18 = 2 ✓

41. The edges of a regular octahedron are to be constructed with tubing that costs $1.85 per foot. If each edge is 3 ft in length, how much will all the tubing required for the project cost?

12 × 3′ = 36′ × $1.85/ft

= $66.60

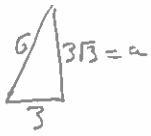


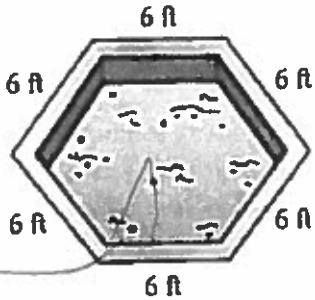
42. Find the surface area of the right rectangular prism if the base has sides 9.4 ft and 5.6 ft and with height 4.5 ft. Round to the nearest tenth.

SA = 2(9.4)(5.6) + 2(9.4)(4.5′) + 2(5.6′)(4.5′)

240.28

43. Samuel is building a flower bed in the shape of a regular hexagon. Each side of the bed is 6 ft long, and the soil is 15 inches deep. How much dirt will be needed to fill the flower bed to the top of the border? Round to the nearest cubic foot.



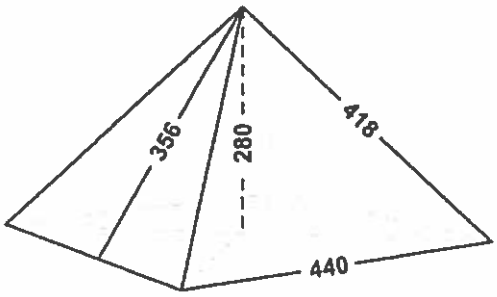


Base
A = ½(6′)(3√3′)(6)
54√3

V = B·h = (54√3 ft²)(1.25 ft) = 117 ft³

116.9134295

44. Find the volume of the Great Pyramid of Giza in Egypt. Assume the base is square with sides measuring 440 ft, and assume the height is 280 ft. Round to the nearest cubic foot.

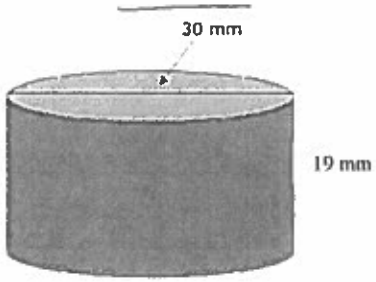


$V = \frac{1}{3} Bh$

$V = \frac{1}{3} \cdot (440')^2 \cdot 280'$

$\approx$ 18,069,333 ft^3

45. Find the surface area of the right circular cylinder. Round to the nearest tenth.



$SA = 2\pi rh + 2\pi r^2$

$= 2\pi(15)(19) + 2\pi(15\text{ mm})^2$

$= 570\pi + 450\pi$

$= 1020\pi \approx 3204.4\text{ mm}^2$

46. A spherical tank with radius 5.25 m is to be filled with a liquid weighing 0.9 grams per cubic centimeter. How many grams of the liquid will the tank hold? Round to the nearest tenth.

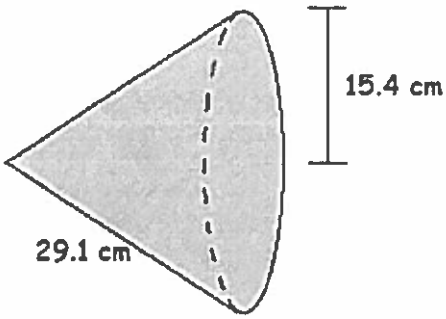
$V = \frac{4}{3}\pi(5.25\text{m})^3 \approx 606.1310326$

$606\text{ m}^3 \cdot \left(\frac{100\text{ cm}}{1\text{ m}}\right)^3 \cdot \frac{0.9\text{ g}}{\text{cm}^3}$

$\approx$ 545,517,929.3 g

545.5 Mg or t

47. Find the surface area of the right circular cone. Round to the nearest tenth.



$SA = \pi r l + \pi r^2$

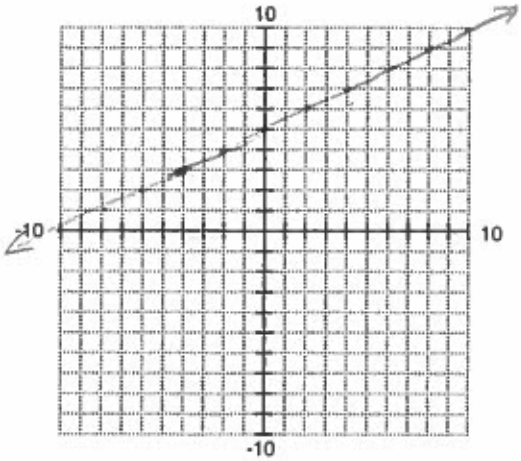
$= \pi(15.4\text{cm})(29.1) + \pi(15.4)^2$

$= 2152.9\text{ cm}^2$

48. Graph the line with slope $\frac{1}{2}$ that passes through the point given by (-4, 3).

Then find its equation in slope-intercept form.

$y = \frac{1}{2}x + 5$



49. For the graph of $x - 2y + 7 = 0$, find the intercepts. Use ordered pairs.

x-intercept (-7, 0)

$x + 7 = 0$

$x = -7$

y-intercept (0, 3.5)

$-2y + 7 = 0$

$7 = 2y$

$y = 3.5$

50. True or False.

(a) The line $y = x$ forms a 45° angle with the positive x-axis. True

(b) The line $y = 5$ is horizontal. True

51. Fill in the blank so that the following pairs of lines are perpendicular.

(a) $y = 3x - 2$ $y = \frac{-1}{3}x + 5$

(b) $y = -4$ $x = 2$

52. Find the equation of the line (in slope-intercept form) parallel to $y = 3x - 5$ and passing through the point given by (-3, 5).

$y - 5 = 3(x - -3)$

$y - 5 = 3x + 9$

$y = 3x + 14$

53. Find the slope-intercept form of the equation of the line perpendicular to the line containing (-2, 0) and (4, -6) which passes through the point midway between them.

$m = \frac{4 - 2}{-6 - 0} = \frac{6}{-6} = -1 \qquad m_\perp = 1$

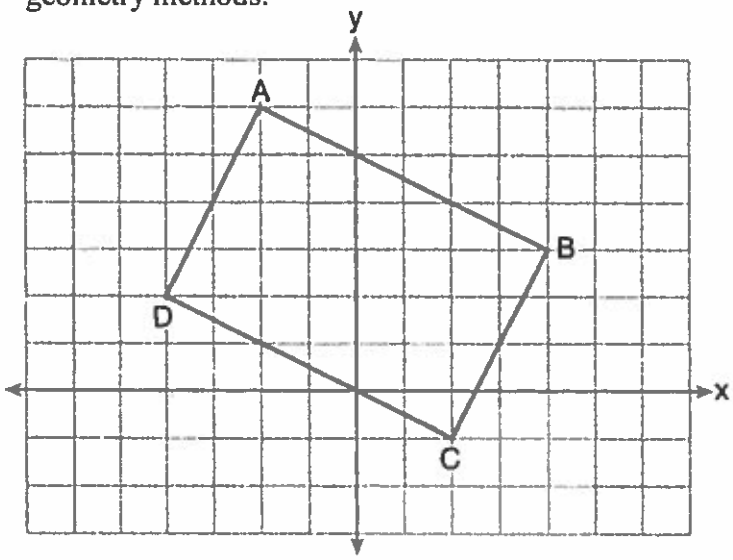
$m = \left(\frac{-2+4}{2}, \frac{0+-6}{2}\right) = (1, -3)$

$y - -3 = 1(x - 1)$

$y + 3 = x - 1$

$y = x - 4$

54. Refer to the diagram. Prove the diagonals of the rectangle are congruent using analytic geometry methods.



A (-2, 6) C (2, -1)

$d_{AC} = \sqrt{(2 - -2)^2 + (-1 - 6)^2}$

$\sqrt{4^2 + (-7)^2}$

$\sqrt{16 + 49} = \sqrt{65}$

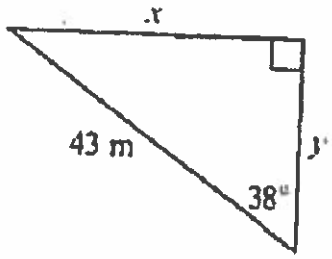
B (4, 3) D (-4, 2)

$d_{BD} = \sqrt{(4 - -4)^2 + (3 - 2)^2}$

$= \sqrt{8^2 + 1^2} = \sqrt{64 + 1} = \sqrt{65}$

$d_{AC} = d_{BD} = \sqrt{65}$ ✓

55. Find x and y to one decimal place.



(a) $x =$ 26.5 m

(b) $y =$ 33.9 m

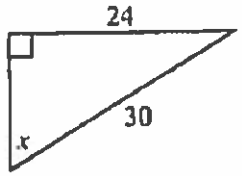
$\sin 38° = \frac{x}{43}$

$x = 43 \sin 38°$

$\cos 38° = \frac{y}{43}$

$y = 43 \cdot \cos 38°$

56. Use a trigonometric ratio to find the measure of the indicated angle to the nearest tenth of a degree.

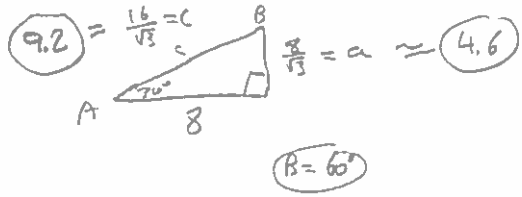


$\sin x° = \frac{24}{30}$

$x = \sin^{-1}(0.8) \approx 53.1°$

57. Solve the given right triangle. Remember that m<C = 90°. Round answers similar to the given measures.

b = 8 cm and m<A = 30°

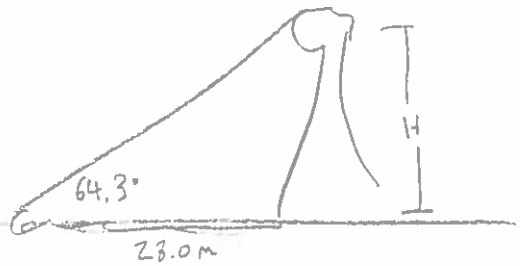


58. A tree casts a shadow of 28.0 meters. The angle of elevation from the sun from the tip of the shadow is 64.3°. What is the height of the tree? Assume level ground, and round to the nearest tenth.

$\tan 64.3° = \frac{H}{28}$

$H = 28 \tan 64.3°$

$H = 58.2$ m



59. List three facts about non-Euclidean geometries that we learned in this course.

(1) The Sierpinski Triangle is probability (dice) - related. / Lots of beauty in fractal geometry.

(2) Fractal antennas would be an interesting application — Something to consider

(3) Spherical geometry has its roots in Babylon several thousand years ago, with great circles as the main construct.